Aging in mean field spin glasses Universality of the arcsine law aging regime

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It is customary in theoretical statistical physics to present glasses as the source of outstanding unsolved problems. At a microscopic level they are strongly disordered, correlated systems that undergo a liquid to solid transition upon appropriate cooling but without any apparent order emerging, and the resulting solid is never observed in equilibrium on laboratory times scales – instead, it undergoes a slow relaxation dynamics with peculiar universal properties that physicists have termed *aging*.
The aging phenomenon opened a wealth of new problems of probability theory in connection with Markov jump processes in highly disordered random environments. Although many of these problems remain unanswered, the case-by-case analysis of several models that began in the early 2000s allowed to isolate a general mechanism that relates aging to the classical arcsine law for stable subordinators through the asymptotic behavior of a partial sum process called *clock process*. This links aging to some of the most classical parts of probability, namely, extreme value theory, the fundamental limit theorems for sums of random variables, and Lévy processes.
In this mini-course, I will explain both this general aging mechanism and the key probability results needed to implement it, starting with a simple model, then on models of increasing difficulties. These are:

- *Trap models on the complete graph*

Proposed by J.P. Bouchaud (1992) as simple phenomenological models for spin glass dynamics, trap models are Markov jump processes that describe thermally activated barrier crossing in random landscapes (random environments) made of i.i.d. heavy tailed "traps". Main examples of microscopic systems that trap models aim to describe are Glauber dynamics on state spaces $\{-1,1\}^n$ reversible with respect to the Gibbs measures associated to random Hamiltonians of mean-field spin glasses, such as

- *The Random Energy Model (REM)* of Derrida (1980), sometimes called the simplest mean field spin glass model, and
- *The family of p-spin SK-models,* $p > 2$ (Derrida, 1985).

We will first implement our aging scheme with the simplest possible Glauber dynamics, the so-called *Random Hopping* dynamics whose transition rates do not depend on the random environment. Although physically unrealistic, the relative simplicity of this choice allows important insights to be gained. We will finally deal with the classical but much harder *Metropolis* dynamics, though only in the case of the REM: nothing is know to date about Metropolis dynamics of the the p-spin SK model.