Cutoff at the "entropic time" for sparse Markov chains

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We discuss convergence to equilibrium for a large class of Markov chains in random environment. The chains are sparse in the sense that in every row of the transition matrix P the mass is essentially concentrated on few entries. Moreover, the entries are exchangeable within each row. This includes various models of random walks on sparse random directed graphs. The models are generally non reversible and the equilibrium distribution is itself unknown. We establish that the mixing time is given by the entropy of the equilibrium distribution divided by the average row entropy of P, and that the chains exhibit the so-called cutoff phenomenon at this "entropic time". As an application, one can consider the case where the rows of P are i.i.d. random vectors in the domain of attraction of a Poisson-Dirichlet law. Our main results are based on a detailed analysis of the weight of the trajectory followed by the walker.