Topological Methods in the Quest for Periodic Orbits

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Resumo/Abstract:
The quest for periodic orbits of dynamical systems - for instance closed geodesics or periodic orbits of particles in a magnetic field - dates back, at least, to the foundational work of Poincar around 1900, followed by work, among many others, by Lusternik-Schnirelmann in the 1920s, Kolmogorov-Arnol'd-Moser around the 1960s, and Rabinowitz and Conley-Zehnder in the early 1980s. The theme of our course is Floer's approach to infinite dimensional Morse theory which revolutionized the field in the second half of the 1980s. Floer combined the Conley-Zehnder approach with Gromov's J-holomorphic curves. This marked a breakthrough in the efforts to prove the Arnol'd conjecture: The number of ONE-periodic orbits of a Hamiltonian vector field on a closed symplectic manifold M is bounded below by the sum of the Betti numbers of M. After introducing necessary elements of symplectic geometry we give the construction of Floer's theory (detecting ONE-periodic orbits of arbitrary energy). Then we present Rabinowitz-Floer theory, a field of intense current research, that detects periodic orbits of fixed energy, but no condition on the period! Finally we indicate how both theories relate to the topology of free loop spaces.

Pré-requisitos: Basic notions of differential geometry: Manifolds, covariant derivative, exponential map, geodesics and Basic notions of functional analysis: Banach and Hilbert spaces, linear operators.