Stochastic algorithms for inverse problems involving PDEs and many measurements

U. Ascher

Vancouver, Canada

Abstract:
Inverse problems involving systems of partial differential equations (PDEs) can be very expensive to solve numerically. This is so especially when many experiments, involving different combinations of sources and receivers, are employed in order to obtain reconstructions of acceptable quality. The mere evaluation of a misfit function (the distance between predicted and observed data) often requires hundreds and thousands of PDE solves. We develop and assess dimensionality reduction methods, both stochastic and deterministic, to reduce this computational burden.

We present in detail our methods for solving such inverse problems for the famous DC resistivity and EIT problems. These methods involve incorporation of a priori information such as piecewise smoothness, bounds on the sought conductivity surface, or even a piecewise constant solution. A more general random subset method is proposed first. We then assume that all experiments share the same set of receivers and concentrate on methods for reducing the number of combinations of experiments, called simultaneous sources, that are used at each stabilized Gauss-Newton iteration. Algorithms for controlling the number of such combined sources are proposed and justified. Evaluating the misfit approximately, except for the final verification for terminating the process, always involves random sampling. Methods for selecting the combined simultaneous sources, involving either random sampling or truncated SVD, are proposed and compared. Highly efficient variants of the resulting algorithms are identified.

Joint work with Farbod Roosta-Khorasani and Kees van den Doel