**L-Functions of Elliptic Curves**

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One of the most fascinating subjects in mathematics is number theory. It has a long and rich history, from the first attempts by the ancient Greeks to understand what numbers are to the modern methods like transcendental number theory and arithmetic geometry. Especially in recent years, the progress has been enormous, and some of the results in number theory found applications not only in other branches of mathematics, but also in cryptography, computer science and physics.

Some of the most important conjectures in number theory concern the so-called L-functions. An L-function is associated to a number-theoretic equation or a system of equations and encodes the most essential information for these equations. Conse-quently, many problems in number theory and geometry, old and new, admit a natural formulation in terms of these L-functions. It su#ces to mention that out of seven Clay Institute Millenium Problems which represent milestones in all of mathematics, two directly concern L-functions (the Birch-Swinnerton-Dyer conjecture and the Riemann hypothesis) and one is strongly related to them (the Hodge conjecture).

The goal of the proposed course is to introduce the students to the arithmetic in-varints associated to elliptic curves and their L-functions, with a particular emphasis on the Birch and Swinnerton-Dyer conjecture and its consequences. It is perhaps the most important unsolved problem remaining in number theory, and it has seen many developments in the past two decades. In particular, one of its consequences, the so-called Selmer parity for elliptic curves over the rationals remained a conjecture for over 40 years and was solved only two years ago. I would like to explain most of the ideas behind its proof, especially that they are quite elementary and seem suitable for the level of this course. I hope that the students will have an opportunity to learn many important tools from modern number theory and generally get a feeling of what is happening in arithmetic geometry and the theory of elliptic curves. I will try and illustrate all presented concepts with many numerical examples.

Tentative structure of the course:
Lecture 1. Elliptic curves and the Mordell-Weil theorem.
Lecture 2. Elliptic curves over finite fields and naive BSD.
Lecture 3. The Birch{Swinnerton-Dyer conjecture and the parity conjecture.
Lecture 4. Parity predictions.
Lecture 5. Local invariants of elliptic curves.
Lecture 5. BSD II and isogeny invariance.
Lecture 6. Parity Conjecture for elliptic curves with an isogeny.
Lecture 7. Finiteness of X implies parity.
Lecture 8. Root numbers (if time allows).