

A maximum principle for Multidimensional BSDEs

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Abstract

We develop the maximum principle for the linear combination of utilities in the multidimensional BSDE setting. In this work utility are of the recursive form i.e it is defined as the solution of BSDE's. We solve the problem using the utility gradient and supergradient approach. The maximum principle which we develop can be used in varied fields of economics and finance. Some of the main application in this paper are from Pareto Optimality and Altruism, which are very well known problems in economics. Previous work have considered either additive utility or recursive utility for 1 dimensional case. We show that the (necessary and sufficient) first-order conditions for the maximization of linear combination of utilities take the form of a forward-backward stochastic differential equation. Under invertibility condition and translation-invariant preferences (a class that includes time-additive exponential utility), the system uncouples. We obtain closed-form solutions for some parametric examples, in the application for Pareto Optimality and Altruism. One of the famous utility forms which we have worked out is "Catching Up with the Joneses", which is very commonly associated with Altruism.

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