A Brownian analogue to Mountford-Prabhakar's theorem

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Burke's theorem [1] states that the departure process of a stationary $M/M/1$ queue with arrivals $Poisson(\lambda)$ has as departure process a $Poisson(\lambda)$. In other terms, $Poisson$ distribution is a fixed point for the $\cdot/M/1$ operator acting on point processes. Mountford-Prabhakar's theorem [4] asserts that, under the ·/M/1 queueing operator, the Poisson distribution is not only a fixed point but also an attractor in a wide class of point processes in the line.

In [5] some analogues of Burke's theorem and further generalizations for Brownian motion were introduced by O'Conell and Yor. The proof of first Burke's theorem analogue backs to Harrison [3], and it relies on weak convergence or some results of Brownian motion itself. In this work we present an overview of both theorems (Burke's and Mountford-Prabhakar's), and we present that Brownian motion is an attractor, under the Brownian queue operator, in a wide class of continuous-valued process and some particular initial conditions (all the tandem queues begin with independent storage, with the stationary distribution of the Brownian queue). This last result turns out to be a continuous analogue to Mountford-Prabhakar's theorem. The coloring coupling which was used to prove the original Mountford-Prabhakar's theorem is no longer avalaible and instead some different coupling techniques are used based on synchronous coupling of Brownian motion, see for example [2] for this topic. We discuss some work in progress related to this setting.

References

[1] P. Burke. The output of a queuieng system. Operations Research, (1956).

[2] K. Burdzy and Z. Chen. Coalescence of synchronous couplings. Probability Theory and Related Fields,(2002).

[3] J. Harrison, R. Williams. On the quasireversibility of a multiclass brownian station. Annals of Probability, (1990).

[4] T. Mountford, B. Prabhakar. On the Weak Convergence of Departures from an Infinite Series of ·/M/1 Queues. Annals of Applied Probability, (1995).

[5] N. O'Connell , M. Yor. Brownian analogues of Burkes theorem. Stochastic Processes and their Applications, (2001).