A partial linearization theorem for three dimensional vector fields and applications

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Abstract: We consider a C^2 vector field X in $R^3$with a critical point at the origin, 0, and real eigenvalues $a < b < 0 < c$ of DX at 0.
We obtain many locally invariant surfaces tangent at 0 to the sum of the eigenspaces of b and c. When these surfaces are C^2, (for instance when $a < 2b$), a well known theorem of Hartman applies to give C^1 linearizations of X on them. In the general case, such invariant surfaces are only $C^{\{1+\alpha\}}$ for some positive alpha. Even in this case, we obtain C^1 linearizations. We then apply this to Poincare maps arising in the Lorenz equations.