The option to delay network investment decision and its impact on the cost-based prices of regulated telecommunications services

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Abstract

This paper proposes a model and methodology for valuing the option to delay network investment decisions and calculating cost-based access prices for regulated telecommunications services. It argues that an option value multiple must be applied to the investment cost component of each network element in order to account for the value of the delay option that is extinguished at the time of investment. Option value multiples are calculated for the investment decision in three main network elements, each representing a different part of the Brazilian fixed telecommunications network, subject to different technological and demand uncertainties. After applying the markup factors, network costs must be assigned to network services on the basis of how much each service uses each network element. The proposed model and methodology can be easily applied on top of existing telecommunications cost studies.

1 Introduction

With the goal of increasing competition, regulatory authorities around the world have adopted cost-based prices for network interconnection and access services. The process of introducing competition has not been easy and many issues have arisen in recent years related to which facilities should be made available by the incumbent carriers, and on what terms and conditions. One of these issues, not yet addressed by regulatory authorities, is the consideration of the value of the option to invest when setting cost-based prices for regulated services.

The need to consider sunk costs in the regulation of tariffs and return on capital in regulated sectors has been recognized since the works of Salinger (1998), Small and Ergas (1999), Alleman and Noam (1999), and Hausman (1999). More recent studies, such as those of Pindyck (2004, 2005), Evans and Guthrie (2006), Hory and Mizuno (2006), Clark and Easaw (2007), Harmantzis and Tanguturi (2007), and Angelou and Economides (2009) have proposed the use of the real options methodology in a variety of applications, including capital budgeting, strategic planning, access pricing and cost modeling. Among the papers in the decision analysis field of cost modeling, two major studies have tried to estimate the impact of real options on the cost-based prices of regulated services using actual telecommunications data: Pindyck (2005) modeled the uncertainty on the demand for basic telephony and ancillary services, while Hausman (1999) modeled the uncertainties on the output price and economic depreciation.

This paper proposes a model and methodology for valuing the option to delay network investment decisions taking into account the demand and technological uncertainties in telecommunications networks. For the costing methodology/approach, it adopts the top-down Long Run Incremental Cost (LRIC) convention. Option value multiples are calculated for the investment decision in three main network elements, each representing a different part of the Brazilian fixed telecommunications network, subject to different technological and demand uncertainties. This methodology can be applied to all network elements typically covered in a LRIC cost study.

This work innovates in many senses. First, it recognizes that different network elements are subject to different technological and demand uncertainties, so that a markup factor is calculated for each main network element. The services provided by different network elements are combined to form a regulated network service (e.g., interconnection, local loop unbundling, etc). Retail costs and revenues are excluded from the markup calculations for (wholesale) network services. Second, the value of the option to invest in each network element is modeled as a function of two stochastic variables: the flow of total variable profit from the hypothetical service provided by the network element, and the cost of new investment in that network element. Third, technological uncertainty is modeled through two different and complementary approaches: one for the technology obsolescence of used equipment and another for the technology evolution of new modern equivalent equipment. Fourth, this study considers that when the used equipment "dies", the incumbent carrier gains a replacement option; that is, the option to invest in the modern equivalent equipment of resized/adjusted capacity.[1]

The uncertainties associated with each network element are modeled through the use of three stochastic processes: the flow of total variable profit (geometric Brownian motion), the depreciation of used asset (Poisson decay process), and the cost of new modern equivalent asset (geometric Brownian motion). They all fit together into a neat and simple model that calculates the option value multiple for each network element. Although this paper focuses on telecommunications, the methodology described here can be applied to other network industries, as well.

2 The Real options model

Network elements form the building blocks of any telecommunications cost study. Each network element is used to provide a unique hypothetical network service, and the services provided by a number of network elements are combined to form a regulated network service (e.g., interconnection, local loop

[1] Prior real options studies have modeled the investment project as specific investments in the telecommunications network – none of them treated the demand and technological uncertainties associated with each network element and none of them excluded retail costs and revenues from the markup calculations. Most of the other studies considered the option value as a function of a single stochastic variable, either output price or demand. None of the prior real options studies simultaneously addressed the risk of technological obsolescence of used equipment and the technological evolution of the modern equivalent asset of resized/adjusted capacity. None of the prior real options studies examined the impact of future replacement options on cost-based access prices.

unbundling, etc), which is either sold to the incumbent carrier's retail business unit or to competitive carriers. Different network elements are subject to different technological and demand uncertainties. For example, switches and transmission equipment are more subject to fast technological substitution than local loop and transmission facilities.

2.1 Model assumptions

Let $\mathrm{NE_i}$ denote one of these network elements (i.e., the network element *i*). To keep the notation simple, the index i has been omitted when no confusion arises. Let Π_t denote the flow of total variable profit at t from the service provided by $\mathrm{NE_i}$; i.e., the flow of total profit ignoring the cost of capital needed to provide this service. The demand for the service provided by $\mathrm{NE_i}$, x_t, is assumed to follow geometric Brownian motion with drift and volatility given by α_x and σ_x, respectively.[2]

$$dx = \alpha_x x dt + \sigma_x x dz_x \qquad (1)$$

It is assumed that the unit contribution margin of $\mathrm{NE_i}$'s hypothetical network service (i.e., the LRIC price minus unit variable cost), p_t, vary exponentially over time, so that $dp = \alpha_p p_t dt$. Therefore, the flow of total variable profit from the service provided by $\mathrm{NE_i}$, $\Pi_t = p_t x_t$, follows geometric Brownian motion (GBM) with drift and volatility respectively given by α_Π and σ_Π.

$$d\Pi = \alpha_\Pi \Pi dt + \sigma_\Pi \Pi dz_\Pi \qquad (2)$$

Where:

$$\begin{cases} \alpha_\Pi = \alpha_x + \alpha_p \\ \sigma_\Pi = \sigma_x \end{cases}$$

The cost of new investment in $\mathrm{NE_i}$ (i.e., the cost of the modern equivalent equipment of resized/adjusted capacity), I_t, depends on two variables: (i) the stochastic price per unit of capacity of $\mathrm{NE_i}$'s modern equivalent asset; (ii) the capacity required to meet the stochastic demand for the service provided by $\mathrm{NE_i}$. It is assumed that the investment in $\mathrm{NE_i}$ displays no economies of scale or modularity effect.[3] If q_t denotes the stochastic price per unit of capacity of $\mathrm{NE_i}$'s modern equivalent asset and x_t denotes the resized/adjusted capacity (as measured in terms of $\mathrm{NE_i}$'s cost driver volume), then $I_t = q_t x_t$. The cost of new investment in $\mathrm{NE_i}$, I_t, is assumed to follow geometric Brownian motion (GBM) with drift and volatility respectively given by α_I and σ_I.

$$dI = \alpha_I I dt + \sigma_I I dz_I \qquad (3)$$

It is assumed that the correlation coefficients between q_t and x_t and between q_t and the market portfolio (M_t) are both positive; that is, $\rho_{q,x} = Corr(\varepsilon_q, \varepsilon_x) > 0$ and $\rho_{q,M} = Corr(\varepsilon_q, \varepsilon_M) > 0$. Let $\rho_{\Pi,I}$ denote the correlation coefficient between ε_Π and ε_I.

[2] The demand for the service provided by a NE can be measured, for example, in terms of minutes of traffic, call setups, access lines, or 2MB circuits. Past historical demand is given by the total cost driver volume measured in a given period.

[3] That assumption is consistent with cost-volume relationship curves that are straight lines passing through the origin. Economies of scale and modularity effects can be introduced at the cost of making the algebra messier.

The market is assumed to be sufficiently complete so that the stochastic fluctuations in Π_t and I_t are spanned by other assets in the economy. Herein, for simplicity of exposition, it is assumed that Π_t and I_t are directly tradable, but it would be sufficient to assume that the risk dynamics of Π_t and I_t, namely the dz_Π and dz_I terms in equations (2) and (3), respectively, could be replicated by some portfolio of traded assets.

The economic depreciation of $\mathrm{NE_i}$ is assumed to be Poisson decay at rate λ $(\lambda > 0)$, so that the event "asset death" follows a Poisson process with rate λ, the asset's lifetime, T_{IO}, has exponential distribution with parameter λ, and the asset's expected value declines exponentially at rate λ. The Poisson risk of asset "death" is assumed to be fully diversifiable, or – in other words – the event "asset death" has correlation zero with the market portfolio. The asset may come to the end of its economic life due to technology, competition and/or mortality factors.

The fundamental equilibrium condition of the capital asset pricing model (CAPM) states that:

$$\begin{aligned} \mu_\Pi &= r + \phi_M \sigma_\Pi \rho_{\Pi,M} \\ \mu_I &= r + \phi_M \sigma_I \rho_{I,M} \end{aligned} \tag{4}$$

Where μ_Π and μ_I are the total expected rates of return of the (directly tradable) assets Π_t and I_t, respectively, r is the discount rate appropriate to riskless cash flows, $\rho_{\Pi,M}$ is the correlation coefficient between Π_t and the whole market portfolio M_t, $\rho_{I,M}$ is the correlation coefficient between I_t and the whole market portfolio M_t, and ϕ_M is the continuous-time market price of risk. Those assets are held by investors only if they provide a sufficiently high return, where part of that return comes in the form of expected capital appreciation (α_Π and α_I), and another part may come in the form of a dividend rate (δ_Π and δ_I).

2.2 *Value of the option to invest*

The value of the installed project (V_t) is a function of the flow of total variable profit $(\Pi_t = p_t x_t)$:

$$V_t = V(\Pi_t) = E[\int_0^{T_{IO}} e^{-\mu_\Pi s} \Pi_{t+s} ds] = \frac{\Pi_t}{(\delta_\Pi + \lambda)}\text{, where } \delta_\Pi = \mu_\Pi - \alpha_\Pi \tag{5}$$

Let $F(\Pi, I)$ denote the value of the option to invest in $\mathrm{NE_i}$. We assume that the investment option is available to the incumbent carrier in perpetuity. Following the steps of contingent claim valuation (see, for example, Dixit and Pindyck, 1994), it can be shown that $F(\Pi, I)$ must satisfy the following partial differential equation:

$$\begin{cases} \tfrac{1}{2}(\sigma_\Pi^2 \Pi^2 F_{\Pi\Pi} + 2\rho_{\Pi,I}\sigma_\Pi\sigma_I \Pi I F_{\Pi I} + \sigma_I^2 I^2 F_{II}) + (r-\delta_\Pi)\Pi F_\Pi + \\ + (r-\delta_I) I F_I - rF = 0 \end{cases} \tag{6}$$

The partial differential equation (6) can be reduced to an ordinary differential equation by making:

$$F(\Pi, I) = IF(\Pi/I, 1) = If(\pi)$$

It follows that $f(\pi)$ must satisfy the ordinary differential equation:

$$\frac{1}{2}(\sigma_\Pi^2 - 2\rho_{\Pi,I}\sigma_\Pi\sigma_I + \sigma_I^2)\pi^2 f''(\pi) + (\delta_I - \delta_\Pi)\pi \; f'(\pi) - \delta_I \; f(\pi) = 0 \tag{7}$$

2.3 Value of the installed project along with all future replacement options

Let $G(\Pi, I)$ denote the value of the installed $\mathrm{NE_i}$ along with that of all future replacement options. First, consider the region of the (Π, I) space where $0 \le \pi < \pi^*$. Following the dynamic programming approach under the risk-neutral equivalent martingale measure Q, it can be shown that $G(\Pi, I)$ must satisfy the following partial differential equation:

$$\begin{cases} \frac{1}{2}(\sigma_\Pi^2 \Pi^2 G_{\Pi\Pi} + 2\rho_{\Pi,I}\sigma_\Pi\sigma_I \Pi I G_{\Pi I} + \sigma_I^2 I^2 G_{II}) + (r - \delta_\Pi)\Pi G_\Pi + \\ + (r - \delta_I) I G_I - (r + \lambda) G + \Pi + \lambda F = 0 \end{cases} \tag{8}$$

Again, the partial differential equation (8) can be reduced to an ordinary differential equation by making:

$$G(\Pi, I) = IG(\Pi/I, 1) = Ig(\pi)$$

It follows that $g(\pi)$, $0 \le \pi < \pi^*$, must satisfy the ordinary differential equation:

$$\begin{cases} \frac{1}{2}(\sigma_\Pi^2 - 2\rho_{\Pi,I}\sigma_\Pi\sigma_I + \sigma_I^2)\pi^2 g''(\pi) + (\delta_I - \delta_\Pi)\pi \; g'(\pi) - (\delta_I + \lambda) g(\pi) + \\ + \pi + \lambda f = 0 \end{cases} \tag{9}$$

Taking the same steps as above for the region of the (Π, I) space where $\pi > \pi^*$, it follows that $g(\pi)$ must satisfy the ordinary differential equation:

$$\begin{cases} \frac{1}{2}(\sigma_\Pi^2 - 2\rho_{\Pi,I}\sigma_\Pi\sigma_I + \sigma_I^2)\pi^2 g''(\pi) + (\delta_I - \delta_\Pi)\pi \; g'(\pi) - \delta_I \; g(\pi) + \\ + \pi - \lambda = 0 \end{cases} \tag{10}$$

The two branches of $g(\pi)$ must meet tangentially at π^*.

2.4 The option value multiple for network investment decision

The value of the option to invest per unit of investment, $f(\pi)$, must satisfy the value-matching and smooth-pasting conditions with $\{g(\pi) - 1\}$ at the investment threshold, π^*. Solving these equations for the free boundary line separating the regions of waiting and investing in the (Π, I) space:

$$\pi^* = (\Pi/I)^* = \frac{\beta_1'}{\beta_1' - 1}(\delta_\Pi + \lambda) \tag{11}$$

Where:

$$\begin{cases} \beta_1' = \frac{1}{2} - \frac{(\delta_I - \delta_\Pi)}{\sigma_T^2} + \sqrt{\left[\frac{(\delta_I - \delta_\Pi)}{\sigma_T^2} - \frac{1}{2}\right]^2 + \frac{2(\delta_I + \lambda)}{\sigma_T^2}} \\ \sigma_T^2 = \sigma_\Pi^2 - 2\rho_{\Pi,I}\sigma_\Pi\sigma_I + \sigma_I^2 \end{cases}$$

When the incumbent carrier invests in $\mathrm{NE_i}$, it gets the installed project valued $V(\Pi^*)$ along with all future replacement options of $\mathrm{NE_i}$. Together, the installed project and all future replacement options value $G(\Pi^*, I)$. If $R(\Pi^*, I)$ denotes he value of all future replacement options, then:

$$G(\Pi^*, I) = V(\Pi^*) + R(\Pi^*, I) = I + F(\Pi^*, I)$$

$$V(\Pi^*) = I + F(\Pi^*, I) - R(\Pi^*, I) = \frac{\beta_1'}{\beta_1'-1} I = m' I \,. \tag{12}$$

The option value multiple for the investment decision in $\mathrm{NE_i}$ is $m' = \beta_1' / (\beta_1' - 1)$. This is the markup factor that must be applied to the investment cost component of $\mathrm{NE_i}$ in order to reflect the true cost of investment, considering both the out-of-pocket cost of investment and the net value of the options involved.

3 The option value multiples calculated for three main investment decisions

In this section, option value multiples are calculated for the investment decision in three main network elements of the Brazilian fixed telecommunications network: Local Access Loop $(\mathrm{NE_A})$; PSTN Host Switch/Duration Sensitive $(\mathrm{NE_B})$; and Host-Host Transmission $(\mathrm{NE_C})$. For that purpose, estimates were calculated for all the parameters in equation (11): the demand drift and volatility estimates were derived from publicly available quarterly time-series data on the number of access lines in service, minutes of local traffic and minutes of transport traffic; the cost of investment drift and volatility estimates were derived from historical prices of modern equivalent assets/equipments; the estimates for $\mu_{\Pi\Delta}$ $(\Delta = \mathrm{A}, B, C)$ were derived from the cost of capital typically used to discount cash flows from the investment in $\mathrm{NE_\Delta}$ $(\Delta = \mathrm{A}, B, C)$; conservative estimates for $\mu_{I\Delta}$ were derived by making $\hat{\mu}_{I\Delta} = \hat{\mu}_{\Pi\Delta}$ $(\Delta = \mathrm{A}, B, C)$;[4] the decay rate estimates were derived from the average lifetimes of similar asset categories found in public and private reports. We performed Dickey-Fuller tests to check for the GBM process fit of $\Pi\Delta_t$ and $I\Delta_t$, and none of these tests were able to reject the assumption that the stochastic variables follow geometric Brownian motion even at mid-high percent level.

If the unit contribution margin of the service provided by each network element is assumed to be constant over time (a common assumption in most telecommunications cost studies – referred to as the static LRIC pricing rule), the option value multiple for the investment decision in $\mathrm{NE_\Delta}$ $(\Delta = \mathrm{A, B, C})$, for $\rho_{\Pi\Delta, I\Delta} \in [0.0,\ 1.0]$, is as follows:

- For the Local Access Loop network element $(\mathrm{NE_A})$, $m_A' \in [1.00,\ 1.05]$,
- For the PSTN Host Switch/Duration Sensitive network element $(\mathrm{NE_B})$, $m_B' \in [1.32,\ 1.41]$,
- For the Host-Host Transmission network element $(\mathrm{NE_C})$, $m_C' \in [1.24,\ 1.33]$.

Figures 1 to 4 show how each option value multiple, m_Δ' ($\Delta = \mathrm{A}, B, C$), changes as a function of the correlation coefficient, $\rho_{\Pi\Delta, I\Delta}$, and - for $\rho_{\Pi\Delta, I\Delta} = 0.5$ - how each option value multiple changes as

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[4] Since the correlations between $q\Delta_t$ and $x\Delta_t$ and between $q\Delta_t$ and the market portfolio (for $\Delta = a, b, c$) are assumed to be positive , the required rate of return $\mu_{I\Delta}$ must be higher than the required rate of return $\mu_{\Pi\Delta}$ (i.e., $\mu_{I\Delta} > \mu_{\Pi\Delta}$ for $\Delta = \mathrm{A}, B, C$). All other things the same, the higher is $\mu_{I\Delta}$, the higher is $\delta_{I\Delta}$, and the higher is the option value multiple.

functions of the volatility parameter of $\Pi\Delta_t$ and $I\Delta_t$, the variable profit drift rate, $\alpha_{\Pi\Delta}$ (which is equal to the demand drift rate if the unit contribution margin is constant), and the investment cost drift rate, $\alpha_{I\Delta}$, all other things held constant.

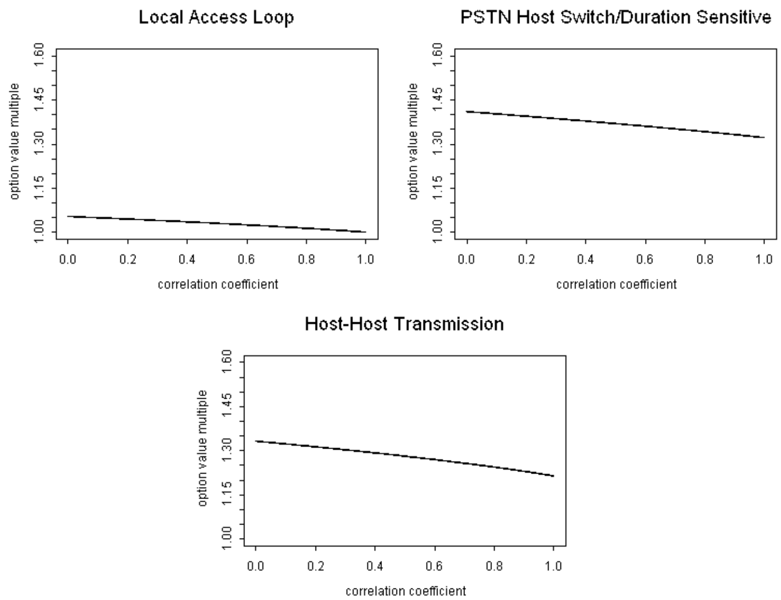

Fig.1 Option value multiple as a function of the correlation coefficient $\rho_{\Pi\Delta,I\Delta}$

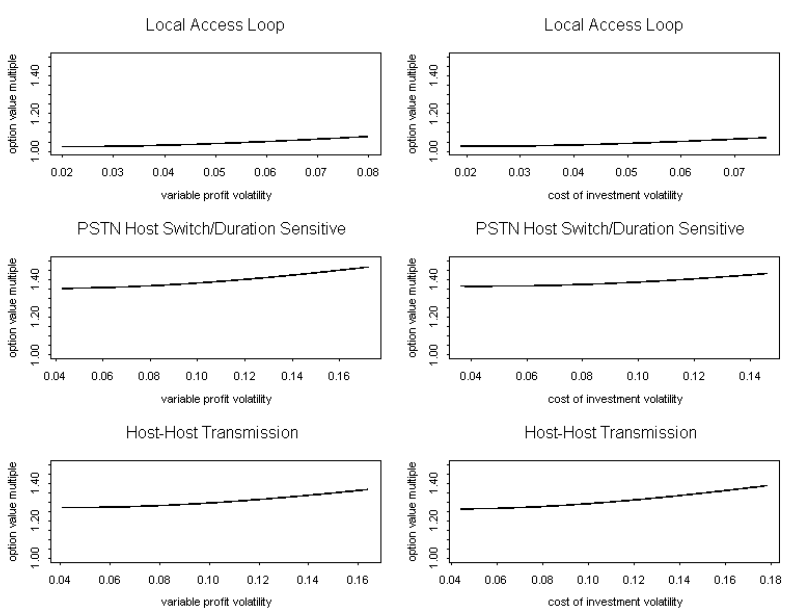

Fig.2 Option value multiple as a function of the volatility parameter of $\Pi\Delta_t$ and $I\Delta_t$

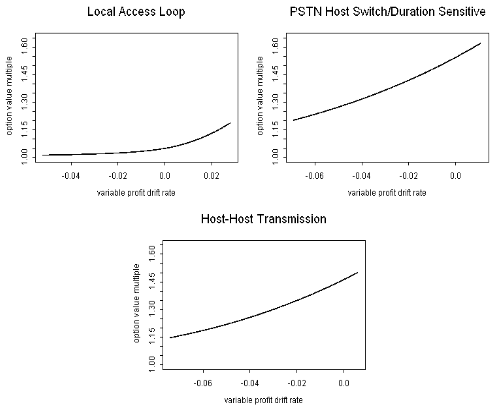

Fig.3 Option value multiple as a function of the variable profit drift rate (for $\hat{\alpha}_{\Pi\Delta} \pm 4\%$).

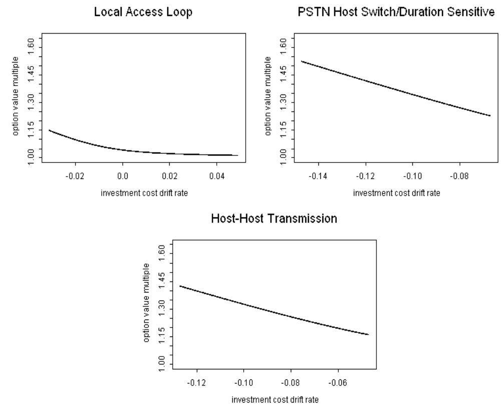

Fig.4 Option value multiple as a function of the investment cost drift rate (for $\hat{\alpha}_{I\Delta} \pm 4\%$).

4 Application of the real options model and methodology

The development of a LRIC cost model typically involves four steps: (i) estimate the investment in new equipment necessary to serve a predetermined level of demand; (ii) estimate the operating expenses required to operate the new equipment; (iii) convert these costs, which are forecast to occur over the life of the investment, into annual costs; and (iv) calculate unitary costs.

Routing factors are used in calculating the demand volumes for each network element (see Appendix A for an illustrative example on how routing factors are calculated). For a given service, routing factors reflect the level of usage of each network element by that service. If $\mathrm{RF}_{i,\Delta}$ denotes the amount of NE_Δ resource consumption by service *i*, the routing factor (demand) volume of NE_Δ $(\Delta = A, B, C)$ in a given year is measured by the routing factor weighted demand on that network element, i.e.:

$$rv\Delta = \text{Routing Factor Volume of } \mathrm{NE}_\Delta = \sum_{\text{all services (i)}} (\text{Demand for Service}_i)\mathrm{RF}_{i,\Delta} \qquad (13)$$

In the final stage of the top-down cost/asset allocation process, the cost and volume data available for each network element are as summarized in Table 2.

Table 2: Cost and volume data of each network element

Network Element	**Cost and Volume Information**	
NE_Δ	(i) Gross Replacement Cost	\$ $g\Delta$
	(ii) Annualized Capital Cost	\$ $y\Delta$
	(iii) Network Operating Expenses	\$ $z\Delta$
	(iv) Allocated Overhead Expenses	\$ $w\Delta$
	(v) Total Operating and Capital Costs	\$ $y\Delta + z\Delta + w\Delta$
	(vi) Routing Factor Volume	$rv\Delta$

Current telecommunications cost models are based on the application of traditional discounted cash flow analysis. The cost-based prices of regulated network services are calculated so as to make the expected present value of the net future cash flows from the service provided by each network element (NE) equal to the cost of investment in that NE. The unitary cost of each network element is calculated by dividing the total operating and capital costs by the annual routing factor volume.

$$\text{Unitary Cost of NE}_\Delta = \$ \frac{(y\Delta + z\Delta + w\Delta)}{rv\Delta}$$

Then, network costs are assigned to services on the basis of how much each service uses each NE.

$$\text{Unitary Cost of Service}_\text{i} = \sum_{\text{all NEs}(\Delta)} (\text{Usage of NE}_\Delta \text{ by Service}_\text{i})(\text{Unitary Cost of NE}_\Delta)$$

In many regulatory environments, the regulated firm is allowed to collect revenue based on the costs faced by a hypothetical efficient replacement firm. In a world where firms have management flexibility to address uncertainties as they are resolved, the true cost of investment faced by the hypothetical efficient replacement firm should include not only the cost of investment in new assets, but also the value of the real options that are extinguished at the time of investment.

Therefore, in order to reflect the true cost of investment, the option value multiple associated with each network investment decision must be applied to the investment cost component (i.e., to the gross replacement cost and/or annualized capital cost) of the respective network element. The *true* unitary cost of investment faced by a hypothetical efficient replacement firm is:

$$\text{True Unitary Cost of NE}_\Delta = \$ \frac{(m_\Delta y\Delta + z\Delta + w\Delta)}{rv\Delta}$$

Where m_Δ is the option value multiple associated with the investment decision in NE_Δ.

The estimation procedure shown in section 3 leads to the option value multiples $m_A' = 1.03$, $m_B' = 1.37$ and $m_C' = 1.28$ (for $\rho_{\Pi\Delta, I\Delta} = 0.5$). Table 3 shows the impact of these markup factors on the average unitary cost of NE_A, NE_B and NE_C, using actual cost and volume data of an incumbent telecommunications carrier, suitably modified to preserve the carriers' anonymity, for the project to build an IP/NGN network in a small metropolitan area.

Table 3: Impact of the markup factor on the average unitary cost of each NE

Network Element	**Average Cost per Unit**		
	before the markup	**after the markup**	**% increase**
Local Access Loop	US$ 298.48 per line (per year)	US$ 302.83 per line (per year)	1.4%
PSTN-Host Switch/ Dur. Sensitive	US$ 0.0035 per minute	US$ 0.0041 per minute	15.1%
Host-Host Transmission	US$ 0.0165 per minute	US$ 0.0182 per minute	9.4%

In the end, the impact of real options on the cost-based price of a regulated network service will depend on the network elements used by that service and the option value multiple calculated for each network investment decision.

The *true* unitary cost of a particular service (Service_i) is:

$$\sum_{\text{all NEs}(\Delta)} (\text{Usage of NE}_\Delta \text{ by Service}_\text{i})(\text{True Unitary Cost of NE}_\Delta)$$

5 Conclusion

There has been a good deal of debate over which markup factor (if any) should be applied to the investment cost component of a network investment decision in order to reflect the value of the *killed* option. Some authors say they are negligible, as in Pelcovits (1999), while others calculate values that are quite significant, as in Hausman (1999) and Pindyck (2005). This paper proposes a model and methodology for valuing the option to delay network investment decisions and calculating cost-based access prices. It shows that the markup values can be negligible for some network elements and quite significant for others.

Working at the network element level allows us to correctly calculate the cost-based price of the hypothetical network service provided by each NE, and thus calculate the cost-based price of any regulated network service. The impact of the option value multiples on the cost-based price of regulated network services will depend on how much each service uses each network element. The proposed model and methodology can be easily applied on top of existing telecommunications cost studies.

Some research perspectives may be highlighted: (i) other sources of uncertainty, such as the uncertainty on discount rates, can be added to the real options model; (ii) statistical models can be built to estimate the drift and volatility parameters of each GBM taking a forward-looking perspective (as opposed to using backward-looking estimates) considering the impact of new entry and/or new technologies possibly not captured using historical data.

Appendix A – An illustrative example of how routing factors are calculated

The example is for local calls, where the telecommunications traffic can take a number of different paths from the originating to the terminating party. Fig. A.1 shows a non-exhaustive list of routes that the traffic for local calls can take (routes 1, 2, 3 and 4). All customers are connected to either a remote or local switch. Depending on the route, different network components are used to process the call. For example, each minute of local call that passes through routes 1, 3 and 4 generates two minutes of host switch local traffic, while each minute of local call that passes through route 2 generates one minute of host switch local traffic.

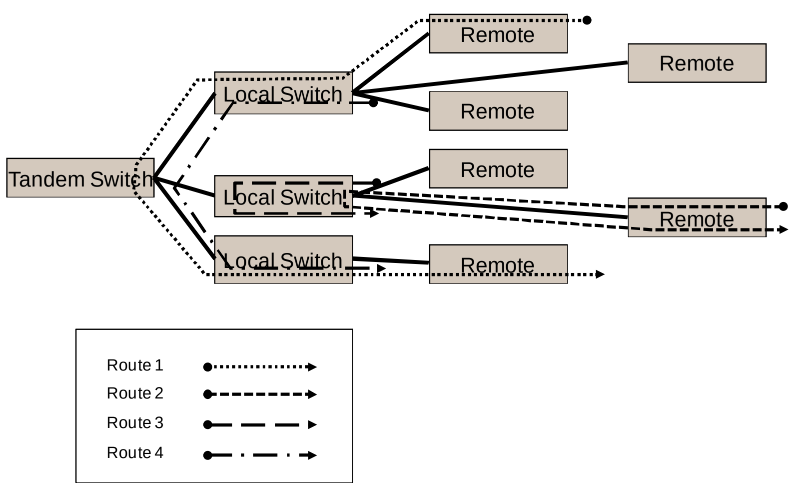

Fig.A.1 Sample of possible routes that can be taken by a local call

The traffic for local calls can take other routes not shown in Fig. A.1 and other network components (e.g., transmission links) are used to process the call that passes through each of these routes. Also, other retail and wholesale services generate telecommunications traffic over these (and other) network components, usually over a different set of possible routes.

For illustrative purpose, suppose that the traffic for local calls can only take routes 1, 2, 3 and 4, and the percentages of traffic that pass through these routes are respectively equal to 15%, 35%, 30% and 20%. If $\mathrm{Service}_1$ is the local call service and NE_B is the PSTN Host Switch/Duration Sensitive network element, the routing factor $\mathrm{RF}_{1,\Delta}$ should be calculated as follows:

$$\mathrm{RF}_{1,B} = 0.15(2) + 0.35(1) + 0.30(2) + 0.20.(2) = 1.65$$

In that simple illustrative example, one minute of local call service demand creates on average 1.65 minutes of host switch local traffic demand (the demand volume for NE_B).

In the end, a routing factor matrix is needed to allocate all network costs to services on the basis of the traffic that each service generates on each network element.

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