Diffeomorphisms of the torus whose rotation sets have non empty interiors

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Abstract:

We consider $C^{1+\epsilon}$ diffeomorphisms of the torus f, either homotopic to the identity or to Dehn twists. We suppose that f has a lift $\widetilde{f}$ to the plane such that zero is an interior point of its rotation set and prove that in this case, there exists a hyperbolic $\widetilde{f}$-periodic point $\widetilde{Q} \in \mathbb{R}^2$ such that $W^u(\widetilde{Q})$ intersects $W^s(\widetilde{Q} + (a, b))$ for all integers (a, b). We also present some consequences of the above result. For example, in the area preserving case,

- $\overline{W^u(Q)}$ coincides with the so called region of instability of f;
- all f-periodic open disks have diameter uniformly bounded from above;
- if f is transitive, then $\widetilde{f}$ is topologically mixing in the plane;
- the rotation vector of the area measure is always in the interior of the rotation set.

The last statement answers a question of P. Boyland. In the general case, we present some uniformity estimates about the displacement of points with respect to vectors in the boundary of the rotation set. These estimates are the main step in proving Boyland's conjecture.