Distribution of Hecke eigenvalues on Hilbert modular groups

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Resumo/Abstract:
Let F be a totally real field, let I be a nonzero ideal of the ring of integers $_F$ of F, let $_0(I)$ be the congruence subgroup of Hecke type of $G = \prod_{j=1}^{d} {}_2()$ embedded diagonally in G, and let be a character of $_0(I)$ of the form $abcd = (d)$, where $d \mapsto (d)$ is a character of $_F$ modulo I.

For a finite subset P of prime ideals not dividing I, we consider the ring ^{I}, generated by the Hecke operators 2, $\in P$ acting on (,)-automorphic forms on G.

Given the cuspidal space $L^{2,\mathrm{cusp}}_{(_0(I)\backslash G,)}$, we let V_ϖ run through an orthogonal system of irreducible G-invariant subspaces so that each V_ϖ is invariant under ^{I}. For each $1 \leq j \leq d$, let $_\varpi = (_{\varpi,j})$ be the vector formed by the eigenvalues of the Casimir operators of the d factors of G on V_ϖ, and for each $\in P$, we take $_{\varpi,} \geq 0$ so that $_{\varpi,}2-$ is the eigenvalue on V_ϖ of the Hecke operator 2

For each family of expanding boxes $t \mapsto_t$ in ^{d}, and fixed an interval J in $[0, \infty)$, for each $\in P$, we consider the counting function

$$N(_t;\, (J)_{\in P}) := \sum_{\varpi,\, _\varpi \in _t \,:\, _{\varpi,} \in J ,\forall \in P} |c^r(\varpi)|^2.$$

Here $c^r(\varpi)$ denotes the normalized Fourier coefficient of order r at ∞ for the elements of V_ϖ, with $r \in'_F \; '_F$ for every $\in P$.

Under some mild conditions on the $_t$, we give the asymptotic distribution of the function $N(_t;\, (J)_{\in P})$, as $t \to \infty$. We show that at the finite places outside I the Hecke eigenvalues are equidistributed with respect to the Sato-Tate measure, whereas at the archimedean places the eigenvalues $_\varpi$ are equidistributed with respect to the Plancherel measure.

As a consequence, if we fix an infinite place l and we prescribe, for fixed intervals $_j$ and J, $_{\varpi,j} \in_j$ for all infinite places $j \neq l$ and

$_{\varpi,} \in J$ for all finite places in P and then allow $|\lambda_{\varpi,l}|$ to grow to ∞, then there are infinitely many such ϖ, and their positive density is as mentioned above. This is joint work with Roelof W. Bruggeman (Utrecht).