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On Coefficient Ideals and Modules

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Abstract

Let $(R, \mathfrak{m})$ be a Noetherian, d-dimensional, local ring and let I an $\mathfrak{m}$-primary ideal of R. The Hilbert-polynomial

$$P_I(n) = \sum_{i=0}^{d} (-1)^i e_i(I) \left(\begin{array}{c} n+d-i-1 \\ d-i \end{array} \right)$$

measures the length of the R-module R/I^n for large n. On the other hand, the integral closure $\overline{I}$ of I is the largest ideal J containing I such that $e_0(I) = e_0(J)$ and the Ratliff-Rush closure $\tilde{I}$ of I is the largest ideal J containing I having the same Hilbert coefficients as I, i.e., $e_i(I) = e_i(J)$, for all $i = 0, \ldots, d$. In this way we have two well-known ideals, $\overline{I}$ e $\widetilde{I}$, associated with with respect to the Hilbert-polynomial of I. Based on these facts, Shah proved in 1991, for $0 \leq k \leq d$, the existence of a unique largest ideals $I_{\{k\}}$, between the ideals $\tilde{I}$ and $\overline{I}$ such that $I_{\{k\}}$ contains I and preserves the first $k+1$ Hilbert-Samuel coefficients $e_0(I), \ldots, e_k(I)$ of I. The ideal $I_{\{k\}}$ is called k**th coefficient ideal** of I.

In this talk we will show how to extend this theory for arbitrary ideals and modules.

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