

# ON A PERTURBATION OF THE BENJAMIN-ONO EQUATION

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## Abstract

We prove using the Fourier restriction norm method that the initial value problem

$$PBO \begin{cases} u_t + uu_x + \beta \mathcal{H}u_{xx} + \eta(\mathcal{H}u_x - u_{xx}) = 0 & x \in \mathbb{R}, \quad t \geq 0, \\ u(x, 0) = \phi(x), \end{cases}$$

where  $\beta, \eta > 0$  and  $\mathcal{H}$  denotes the usual Hilbert transform, is locally well-posed in the Sobolev space  $H^s(\mathbb{R})$  for any  $s > -1/2$  and globally well-posed in  $H^s(\mathbb{R})$  when  $s \geq 0$ . We also prove that one cannot solve this Cauchy problem by a Picard iterative method implemented on its integral formulation in the Sobolev spaces  $H^s(\mathbb{R})$ , with  $s < -1$ . This equation was first introduced by H. H. Chen and Y. C. Lee in [1] to describe fluid and plasma turbulence. Several authors have studied this equation from a numerical standpoint, e.g. H. H. Chen, Y. C. Lee and S. Qian in [2, 3], and B. -F. Feng and T. Kawahara, in [4].

## References

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