Three-dimensional Navier-Stokes simulations of Hele-Shaw flows using finite-difference schemes

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The formation of viscous fingers in Hele-Shaw cells is an instability associated with viscosity differences across the interface between two moving fluids [1-3]. Traditionally, due to its analogy to porous media flows, such displacements have mostly been modeled by Darcy's law, which can be derived by gap-averaging the three-dimensional Stokes equations.

It is clear that the Darcy-based modeling approach has been very successful in reproducing several important phenomena that had been observed experimentally for miscible Hele-Shaw displacements. For example, the early linear stability investigations by Tan & Homsy [4] show that increasing unfavorable viscosity ratios result in larger growth rates and shorter wavelengths of the most unstable mode, and their subsequent nonlinear simulations [5] were able to reproduce the tip-splitting phenomenon. Nevertheless, Fernandez et al. [6] experimentally determined dispersion relations for miscible Rayleigh-Taylor instabilities in vertical Hele-Shaw cells. The corresponding linear stability analysis by Graf et al. [7], based on the three-dimensional Stokes equations, observes good agreement with the experimental growth rates across five orders of magnitude in the Rayleigh number. These authors also compare the Stokes-based dispersion relations with corresponding results derived from a gap-averaged Darcy approach, and they find reasonable agreement only for moderate Rayleigh numbers. For large values of this parameter, the three-dimensional flow structure within the gap becomes important, so that averaging across the gap can no longer capture the dominant physical mechanisms. Additional Stokes-based linear stability results for variable viscosity Hele-Shaw displacements were discussed by Goyal & Meiburg [8,9], Schafroth et al. [10], and Goyal et al. [11], showing similar discrepancies between Stokes-based and gap-averaged results.

The present investigation intends to focus on the evolution of quasisteady fingers and their subsequent instabilities in the nonlinear regime through direct numerical simulations of the three-dimensional variable viscosity Navier-Stokes equations coupled with a convection-diffusion equation of a concentration field. The numerical approach follows Rai & Moin [12] and uses finite differences in a three-step hybrid Runge-Kutta/Crank-Nicolson discretization rendering the overall accuracy second order in time. A factorization approximation is applied to the implicit diffusive terms of the governing equations. The alternating direction implicit (ADI) method employed maintains the overall second order accuracy of the method, reduces memory requirements, and allows to solve three tridiagonal systems instead of inverting a large sparse matrix [13]. Most of the discretization employs second order central differences on a uniform staggered grid, but the derivatives of the convective terms are calculated using the fifth order WENO. This fractional-step method takes advantage of a projection method to solve for the corrector term in each step of the RK iteration. The resultant Poisson equation is solved using cosine transformations.

We will show the results of a simulation that reached 665.4 million grid points running on 256 processors, and the formation of a hidrodynamic instability that cannot be described by two-dimensional laws [14].

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