New Bounds for the hard-core model on the square lattice

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Abstract :
The hard-core model has received much attention in the past couple of decades as a lattice gas model with hard constraints in statistical physics, a multicast model of calls in communication networks, and as a weighted independent set problem in combinatorics, probability and theoretical computer science.

In this model, each independent set I in a graph G is weighted proportionally to $\lambda^{|I|}$, for a positive real parameter λ. For large λ, computing the partition function (namely, the normalizing constant which makes the weighting a probability distribution on a finite graph) on graphs of maximum degree $d\ge 3$, is a well known computationally challenging problem. More concretely, let $\lambda_c(T_d)$ denote the critical value for the so-called uniqueness threshold of the hard-core model on the d-regular tree; recent breakthrough results of Dror Weitz (2006) and Allan Sly (2010) have identified $\lambda_c(T_d)$ as a threshold where the hardness of estimating the above partition function undergoes a {\em computational} transition.

We focus on the well-studied particular case of the square lattice Z^2, and provide a new lower bound for the uniqueness threshold, in particular taking it well above $\lambda_c(T_4)$. Our technique refines and builds on the tree of self-avoiding walks approach of Dror Weitz, resulting in a new technical sufficient criterion (of potential wider interest) for establishing strong spatial mixing (and hence uniqueness) for the hard-core model. Our results also imply a fully polynomial deterministic approximation algorithm for approximating the partition function and rapid mixing of the associated Glauber dynamics.

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