

## On the multigraded Betti numbers of an edge ideal

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### Abstract:

We will study how algebraic properties of an edge ideal can be related to the combinatorics of the underlying graph. More concretely, given a simple graph  $G$  and  $I(G)$  the edge ideal associated to  $G$ , we will see how the multigraded Betti numbers and the Castelnuovo-Mumford regularity of  $I(G)$  can be related to the structure of the graph  $G$ . We will first discuss some general statements on the shape of the Betti diagram of edge ideals. Then, we will recall a classical result of Fröberg that characterizes edge ideals of regularity 2 in a combinatorial way. When the regularity is strictly bigger than 2, we can moreover determine the first step  $i$  in the minimal graded free resolution where nonlinear syzygies first appear and show that, at this step  $i$  of the minimal resolution, the highest degree of a minimal generator is  $i + 3$ . The value of the corresponding Betti number  $\beta_{i,i+3}$  can also be determined in terms of the combinatorics of the associated graph. Our main results show how similar statements hold for edge ideals of regularity 3 when the associated graph is bipartite. This is a joint work with Oscar Fernández-Ramos.