Toroidal automorphic forms and the Riemann hypothesis

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Resumo / Abstract:
A formula of Erich Hecke in an article from 1917 laid a connection between a sum of values of the Eisenstein series E(-,s) with the Riemann zeta function \zeta(s). We call an automorphic form toroidal if the corresponding sum (or integral in its adelic formulation) vanishes for all right translates. The importance of this definition lies in a reformulation of the Riemann hypothesis in terms of the space of toroidal automorphic forms as observed by Don Zagier. Namely, the Eisenstein series E(-,s) lies in a tempered representation if and only if s has real part 1/2, and by Hecke's formula, E(-,s) is toroidal if s is a zero of the Riemann zeta function.

We review this connection in detail and give an overview of what is known about the space of toroidal automorphic forms. In particular, the space of toroidal automorphic forms is completely determined by the zeroes of Hecke L-functions (in the Eisenstein case) and Artin L-functions of cuspidal representations (in the case of cusp forms). We will explain some methods of this theory and show, if time allows, how the Riemann hypothesis can be proven via these lines in the case of some function fields.