Higher-order automorphic forms

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The theory of higher-order automorphic forms emerged about ten years ago in the study of distribution of modular symbols and (independently) in the investigation of questions in percolation theory. Since then, the theory has developed rapidly to become an established subject within the area of automorphic forms and their applications. It has attracted the active interest of various world-leading researchers e.g. Bruggeman, Deitmar, Goldfeld and Knopp, and links with other areas of number theory, such as multiple Dirichlet series and L-functions are increasingly manifesting themselves.

We propose to present a mini course on the foundations, achievements up to now and future perspectives of the theory of higher-order automorphic forms. The main objectives of the mini-course will be: to describe the current state-of-the-art and the emerging techniques and applications of the field to highlight the outstanding open problems of the subject and to encourage the active participation of the participants in its future development.

The structure of the mini-course we propose consists of 6 lectures each of duration 1h 30m

Lecture 1: General introduction to the theory of modular forms. The main aim of this lecture is to provide the necessary background to participants whose primary expertise is in other areas and to highlight the approach we will be taking to the subject. Topics include the definitions and important examples of modular and automorphic forms, basic facts about the linear, ring and inner-product structure of the space they form, Hecke operators an introduction to L-functions of cusp forms and examples of the outstanding questions in the subject.

Lecture 2: Background and introduction to the theory of higher-order automorphic forms.
In this lecture the motivation for the introduction of higher-order forms will be discussed with special emphasis on the theory of Eisenstein series modified with modular symbols. We then give the definitions of the various versions of higher-order forms that have emerged in applications. A first classification theorem supplying several types of important examples will be proved.

Lecture 3: The linear structure of the theory of the space of second- and higher-order modular forms Explicit bases for the space of (holomorphic) second-order modular forms will be constructed. This will provide the opportunity to discuss the way spectral and other modern analytic techniques can be exploited in the field. The analogous construction for the case of orders higher than two will also be outlined.

Lecture 4: L-functions and representation-theoretic approaches.
The aim of this lecture is to associate L-functions to second-order automorphic forms.The beginnings of a representation-theoretic approach will be presented and in this context a

Hecke theory will be proposed. An inner product structure compatible with this Hecke action will be further be discussed from two different perspectives.

Lecture 5: Geometric and cohomological aspects of higher-order forms.
An interpretation of higher-order forms in terms of Chen's iterated integrals will be given. An approach based on vector bundles will further be outlined. We will further discuss cohomological interpretations of higher-order automorphic forms, including an introduction to higher-order cohomology.

Lecture 6 Recent developments.
This last lecture will review the latest developments in the area and discuss possible future directions of the subject. Topics include higher-order automorphic forms on the universal covering group, an introduction of techniques based on families of automorphic forms, the connections with the theory of multiple Dirichlet series, applications to Percolation Theory and a discussion of other developments that may occur between now and the time of the mini-course.