EFFICIENT MONTE CARLO FOR DISCRETE VARIANCE DERIVATIVES

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Motivated by the ongoing growth in the market for contingent claims on realized variance we develop an efficient numerical algorithm for their valuation. We consider an arbitrary European claim on discretely recorded realized variance and argue, through an example, that its value can be substantially different from that obtained by assuming continuously recorded variance. Therefore, we model daily returns as arising from a discrete time stochastic volatility model,

$$\begin{aligned} \frac{S_{i+1}-S_i}{S_i} &= rdt + \sigma(S_i)\sqrt{V_i}h(Z_{i+1}) \\ \frac{V_{i+1}-V_i}{V_i} &= \beta(V_i)dt + \gamma(V_i)[\rho h(Z_{i+1}) + \sqrt{1-\rho^2}W_{i+1}], \end{aligned}$$

which includes discretizations of the popular Heston, Hull-White and SABR models as special cases. A non-linear choice of h leads to models with non-Gaussian conditional returns and heavy tails. We consider, as an example, the double exponential model of Kou. We evaluate a payoff G defined on realized variance

$$G(RV) = G(\sum_{i=0}^{N-1}(\frac{S_{i+1}-S_i}{S_i})^2).$$

The price of a contingent claim can be represented as the value of the expected payoff, which in this case becomes a high dimensional integral because it depends on the full realization of the stochastic vectors Z, W. Deterministic numerical integration is unfeasible due to the dimensionality of the problem, and naive Monte Carlo is potentially very noisy.

We construct a price estimator through the combination of deterministic integration along two privileged variables, followed by many Monte Carlo simulations, each conditional on a particular value of the integration variables. These variables are the norm of the Z and the path average of the process V, and are chosen because their variability captures a large fraction of the variability of realized variance. In this regard, we obtain a theoretical result that quantifies the efficiency gain of the proposed method, in terms of the reduction in variance of the new estimator relative to naive Monte Carlo.

The practical implementation of the method requires replacing the path average of V (which typically has an unknown density) by a linear combination of the components of W (therefore

Gaussian) that best captures the variability of the path average of V. This is identified in the literature as the optimal stratification direction.

The numerical algorithm begins by integrating deterministically over the norm of Z, distributed as a χ^2 random variable, and over an independent univariate Gaussian density. The latter is interpreted as a shock along the stratification direction for the average of V. The algorithm then samples all the components of W (conditional on the value of the stratified variable) and the components of Z (conditional on the norm of Z). In this manner Z and W are sampled without any approximation. The option payoff is also computed exactly based on Z and W therefore the proposed method is exact.

Numerical experiments under the Hull-White and Heston models show that, for realistic computational budgets, there is substantial variance reduction with respect to a standard Monte Carlo algorithm.