**Realization of Finite Groups by 3-Nets in a Projective Plane over a Field**

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In a projective plane $PG(2,\mathbb{K})$ over a field $\mathbb{K}$ of characteristic $p \geq 0$, a 3-net consists of three pairwise disjoint classes of lines such that every point incident with two lines from distinct classes is incident with exactly one line from the third class.

A 3-net is said to realizing a group $(G, .)$ when it is coordinatized by $G$. If A, B, C are the classes, then the meaning of this condition is as follows: there exists a triple of bijective maps from G to (A, B, C), say $\alpha : G \to A, \beta : G, \to B, \gamma : G \to C$ such that $a \cdot b = c$ if and only if $\alpha(a), \beta(b), \gamma(c)$ are three collinear points, for any $a, b, c \in G$.

Key examples arise naturally in the dual plane of $PG(2,\mathbb{K})$. An infinite family, due to Yuzvinsky, arises from plane cubics and comprises 3-nets realizing cyclic and direct products of two cyclic groups. Another infinite family, due to Pereira and Yuzvinsky, comprises 3-nets realizing dihedral groups. In [2], we give a complete classification of 3-nets realizing a finite group. If $p = 0$, we prove that there is no further infinite family and list all possible sporadic examples. If $p > |G|$, the same classification holds true apart from three possible exceptions: $A_4$, $S_4$ and $A_5$. The small groups $C_3 \times C_3$, $C_2 \times C_4$ and $A_4$ need specific methods and are studied in the paper [3].

In this talk, we present key examples and some connection to the coset intersection problem for irreducible plane cubics, [1].

# References

[1] G.Korchmaros, N. Pace, Coset Intersection of Irreducible Plane Cubics, to appear in Designs, Codes and Cryptography, 2013.

[2] G. Korchmaros, G. Nagy, N. Pace, 3-nets realizing a group in a projective plane, preprint arXiv:1104.4439.

[3] G. Nagy, N. Pace, On small 3-nets embedded in a projective plane over a field, Journal of Combinatorial Theory, Series A, Volume 120, Issue 7, 1632-1641, September 2013.