Towards a Mathematical Theory of Complex Systems
Reducing Complexity, Multiscale Aspects and Applications

Bellomo Nicola
Department of Mathematics, Politecnico di Torino, Italy
http://calvino.polito.it/fismat/poli

This Lecture is devoted to the development of mathematical tools for the modeling, qualitative analysis and simulations of complex systems in life and human sciences. Namely of systems of many living individuals interacting in a non-linear manner. As known, it is very difficult to understand and model these systems based on the sole description of the dynamics and interactions of a few individual entities localized in space and time. Moreover, interactions are not additive and their modeling should take into account the ability of the interacting entities to develop specific strategies based on the states and localization of all interacting entities. Looking at living systems, definitely the most sophisticated class of complex systems, two main questions can be naturally posed:

Do complex living systems exhibit common features?

Are the analytic and computational tools offered by mathematics able to capture, in the modeling approach, the above mentioned common features?

The first part of this lecture reports about recent developments of the, so called, kinetic theory for active particles [1],[2], which provides some answers to the above questions. The second part focus on on multicellular systems and genetic diseases [3]. The third part deals with the derivation of macroscopic biological tissue models from the underlying description delivered at the cellular scale. The derivation leads to classical and modified Keller and Segel type models [4]]. Finally, it is shown how the approach can be extended to the modeling of crowds and swarms [5].

[1] N. Bellomo, **Modelling Complex Living Systems - A Kinetic Theory and Stochastic Game Approach**, (Birkäuser, Boston, 2008).

[2] N. Bellomo, C. Bianca, and M. Delitala, Complexity analysis and mathematical tools towards the modelling of living systems, *Physics of Life Review*, **6** (2009) 144-175.

[3] N. Bellomo, C. Bianca, **Towards a Mathematical Theory of Multiscale Complex Biological Systems**, Editor World Scientific, Series in Mathematical Biology and Medicine (2010).

[4] N. Bellomo, , A. Bellouquid, J. Nieto, and J. Soler, Multiscale biological tissue models and flux-limited chemotaxis from binary mixtures of multicellular growing systems, *Math. Models Methods Appl. Sci.*, **20** (2010).

[5] N. Bellomo and C. Dogbé, , On The Modelling of Traffic and Crowds - A Survey of Models, Speculations, and Perspectives, SIAM Review, (2010), to appear.