

Path-Dependence Properties of Leveraged Exchange-Traded Funds: Compounding, Volatility and Option Pricing

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In the first part of this talk, we give an exact formula linking the price evolution of a leveraged ETF (LETF) with the price of its underlying ETF. We test the formula empirically on historical data for 56 leveraged funds (44 double-leveraged, 12 tripleleveraged) using daily closing prices. The results indicate excellent agreement between the formula and the empirical data. The formula shows that the evolution of the price of an LETF is sensitive to the realized volatility of the underlying product. The relationship between an LETF and its underlying asset is therefore “path-dependent.”

The second part of the talk focuses on the relationship between options on LETFs and options on the underlying ETFs. The main result shows that an option on an LETF can be replicated by a basket of options on the underlying ETF, after a suitable choice of strikes and notionals. In particular, we obtain a new, relative-value, model for pricing LETF options. The derivation makes strong use of the path-dependency result of Part I. As a consequence, we derive a simple non-parametric formula which links the volatility skew of an LETF with the volatility skew of the underlying ETF.

We validate the theory empirically by showing that the model prices for options on LETFs are in excellent agreement with actual mid-market prices observed in markets. The empirical study was carried out on two LETFs linked to the S&P 500 index (one double-leveraged, one reverse-double-leveraged). The issue of vega-hedging options on LETFs with options on the underlying ETFs is also examined.