Dynamics of dissipative polynomial automorphisms of C^2

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Two-dimensional complex dynamics displays a number of phenomena that are not observable in dimension one. However, if f is moderately dissipative then there are more similarities between the two fields. For instance, dynamics on an invariant Fatou component admits a nearly complete description:

Theorem 1. Any invariant component D of the Fatou set is either an attracting basin or parabolic basin, or the basin of a rotation domain (Siegel disk or Herman ring).

Theorem 2. In the first two cases, D contains a "critical point".

In complex and real one-dimensional world, structurally stable maps are dense. In dimension two this fails because of the Newhouse phenomenon caused by homoclinic tangencies. Palis conjectured that in the real two-dimensional case this is the only reason for failure. We prove a complex version of this conjecture:

Theorem 3. Any moderately dissipative polynomial automorphism of C^2 is either ``weakly stable" or it can be approximated by a map with homoclinic tangency.

Theorem 1 is joint with Han Peters, the rest is joint with Romain Dujardin.