General Theory of Geometric Lévy Models for Dynamic Asset Pricing

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Abstract / Resumo:
The theory of Lévy models for asset pricing simplifies considerably if one takes a pricing kernel approach, which enables one to bypass market incompleteness issues. The special case of a geometric Lévy model (GLM) with constant parameters can be regarded as a natural generalisation of the standard geometric Brownian motion model used in the Black-Scholes theory. In one dimension, once the underlying Lévy process has been specified, the GLM is characterised by four parameters: the initial asset price, the interest rate, the volatility, and a risk aversion factor. The pricing kernel is given by the product of a discount factor and the Esscher martingale associated with the risk aversion parameter. The model is fixed by the requirement that for each asset the product of the asset price and the pricing kernel should be a martingale. In the GBM case, the risk aversion factor is the so-called market price of risk. In the GLM case, this interpretation is no longer generally valid; instead, the excess rate of return is given by a nonlinear function of the volatility and the risk aversion. We show that for positive values of the volatility and the risk aversion the excess rate of return above the interest rate is positive, and is monotonically increasing with respect to these variables. In the case of foreign exchange, Siegel's paradox implies that it is possible to construct FX models for which the excess rate of return (above the interest rate differential) is positive both for the exchange rate and the inverse exchange rate. We show that this condition holds for any GLM for which the volatility exceeds the risk aversion. Similar results hold in the case of a multiple-asset market driven by a vector of independent Lévy processes. (Co-authors: Dorje Brody, Brunel University, and Ewan Mackie, Imperial College Business School)