MAXIMUM LEBESGUE EXTENSION OF CONVEX RISK MEASURES

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Our basic interest is how far the notion of convex risk measure can be extended beyond the space of bounded random variables. More specifically, given a convex risk measure ρ^0 which is defined with a "nice regularity property" on L^∞, we want to extend it as far as possible preserving the regularity, and to know what is a maximal space to which such a "regular extension" is possible. The regularity property that we take up here is the "Lebesgue property", an analogue of the "dominated convergence" in measure theory (but generally stronger than the Fatou property), and many of concrete examples of risk measures have this property on L^∞. Based on a simple observation, we first construct a space of random variables beyond which ρ^0 can not has a regular extension, then show that (1) the space thus constructed is an "order-continuous Banach lattice" with a gauge norm, (2) ρ^0 indeed has a regular extension to this space, and (3) such an extension is unique. In particular, this Banach lattice is maximum among all solid spaces random variables which accommodate a regular extension of ρ^0. We then characterize this space in connection to Orlicz-type spaces and uniform integrability.

As an application, we give a generalization of Jouini-Schachermayer-Touzi's characterization of Lebesgue property [Adv. Math. Econ. vol.9, pp49-71]. Our extension is valid on any solid space of random variables containing the constants, and is universal in the sense that it does not rely on any topological structure of the particular space.