

# The smile in stochastic volatility models

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In this talk, we consider general stochastic volatility models and derive the smile of implied volatility at second order in the volatility of volatility using perturbation techniques. At this order, the smile is cubic in the log-moneyness. It depends on only three numbers which synthesize respectively spot-variance, variance-variance, and spot-skew covariances. We compare with the true smile for the Bergomi model with two factors on the variance curve. The agreement is excellent at first order for the ATM skew, and at second order for the ATM implied volatility, up to typical values of the volatility of volatility (say, 200% in the equity market). We also derive a general relationship between ATM skew and skewness of the distribution of the log-spot, at first order in the volatility of volatility.

## References

- [1] Bergomi L., *Smile Dynamics 2*, Risk Magazine, pages 67-73, October 2005.
- [2] Backus D., Foresi S., Li K. and WU L., *Accounting for Biases in Black-Scholes*, unpublished.
- [3] Lee R., *The moment formula for implied volatility at extreme strikes*, Stanford University and Courant Institute, 2002.