DYNAMICAL BEHAVIOR OF ERGODIC MEASURE ALONG WEAK DIRECTION

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By the Pesin theory, the lamination of the strongest invariant manifolds of an ergodic measure is always absolutely continuous. But for the weak/center direction, the dynamical property may be very different. For example, there are partially hyperbolic diffeomorphisms whose center foliations are not absolutely continuous. Such kind of example was first observed by Katok: the diffeomorphism is partially hyperbolic with vanishing center exponent, moreover, there is a full Lebesgue measure subset which intersects every center leaf with finitely many (even unique) points. This phenomenon is called Fubini's nightmare.

Soon it was observed by Ruelle and Wilkinson that, for a volume preserving partially hyperbolic diffeomorphism, if the volume of every center leaf does not expand, then the existence of positive center exponent implies the Fubini's nightmare. For example, we can consider diffeomorphisms with positive center exponent, which are $\mathcal{C}^1$ perturbations of time-one map of a hyperbolic geodesic flow, or perburbations of a skew product map: $A \times Id \in \mathrm{Diff}(T^2 \times S^1)$, where A is an Anosov diffeomorphism over T^2. This result has been generalized by Viana, Yang to $\mathcal{C}^1$ dissipative diffeomorphisms.

It was unknown that when the center has expansion, how the dynamics (along the center direction) look like. In this talk, we will study 3 dimensional diffeomorphisms which are in the same isotopy class with A_0, where A_0 is a linear Anosov diffeomorphism over T^3 with 3 exponents $\lambda_1 < 0 < \lambda_2 < \lambda_3$.

Theorem 0.1. *Let f be in the same isotopy class with A_0, and μ an ergodic measure of f with metric entropy larger than $\log \lambda_3$. Then there is a μ full measure subset Λ, such that Λ intersects μ almost every center leaf with uncountable number of points. Moreover, the center foliation is not μ measurable.*

Remark 0.2. *If the diffeomorphism f in above theorem is volume preserving, and whose center exponent is largger than λ_2, then the center foliation is non-atomic and non-absolutely continuous. The existence of such foliation was asked by Wilkinson, and the first example was given by R. Varão.*

Remark 0.3. *The lower bound $\log \lambda_3$ of metric entropy is rigid. There is an example of Ponce, Tahzibi and Varão, shows that, when the metric entropy of an ergodic measure equals to $\log \lambda_3$, the center foliation can be measurable, and there is a full measure subset which intersects every center leaf with at most one point.*

Suppose f is a partially hyperbolic diffeomorphism in the same isotopy class of A_0, there is a semiconjugation π_f between f and A_0. The following theorem generalizes a result of Ures for maximal measures:

Theorem 0.4. *Let f be a partially hyperbolic diffeomorphism in the same isotopy class of A_0. Then $(\pi_f)_*$ induces a 1-1 map between the ergodic measures of f and A_0 with metric entropy larger than $\log \lambda_3$.*

Remark 0.5. *The above theorem shows that f and A_0 have the same kind of ergodic measures with large entropy.*

Our proof makes use of dimension theory for the center direction.

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