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SHARP ILL-POSEDNESS RESULTS FOR THE BENNEY SYSTEM

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1 Introduction

We consider the Benney system, given by the coupled equations:

$$\begin{cases} i\partial_t u + \partial_x^2 u = & \alpha uv + \beta|u|^2 u, \quad x, t \in \mathbb{R}, \\ \partial_t v + \lambda \partial_x v = & \gamma \partial |u|^2, \quad u(x,0) = u_0(x) \qquad v(x,0) = v_0(x), \end{cases} \tag{1.1}$$

where (u_0, v_0) is considered in the classical Sobolev spaces $H^k(\mathbb{R}) \times H^s(\mathbb{R})$.

This system appears in general theory of water wave interaction in nonlinear medium and was introduced by Benney in [1] and [2].

We discuss ill-posedness issues for the initial value problem. More precisely, we show that the flow map associated to the system is not C^2 for several Sobolev's exponents (k, s) outside of the well-posedness regime.

We follow the arguments used in [5] to show ill-posedness results for the Zakharov system. Moreover, some of these results can be adapted to the Schrödinger-Debye system.

2 Results

The most general result about local well-posedness for the system (1.1) was established in 1997 by Ginibre, Tsutsumi and Velo in [4]. In this work the authors showed the following theorem:

Theorem 1. *The Benney System (1.1) is locally well-posed with initial data $(u_0, v_0) \in H^k(\mathbb{R}) \times H^s(\mathbb{R})$ satisfying*

$$-\frac{1}{2} < k - s \leq 1 \quad \textit{and} \quad 2k \geq s + \frac{1}{2} \geq 0.$$

This result was obtained using fixed point arguments in Bourgain spaces.

Later, in [3], Corcho showed that the local result in $L^2 \times H^{-1/2}$, obtained in [4], is the best possible in the line $s = k - 1/2$ for the focusing situation $(\beta < 0)$. More precisely, he proved the following result:

Theorem 2. *The focusing Benney System (1.1) does not have flow continuous uniformly in $H^k(\mathbb{R}) \times H^s(\mathbb{R})$ provided $-\frac{1}{3} \leq k < 0$ and $k(2s+1) \geq -1$.*

The method used was based on the article [6] of Kenig, Ponce and Vega, where the authors showed the instability of the flow for certain traveling wave solutions in the context of the Schrödinger equation.

In this work we generalizes the theorem proved in [3] for a larger region of exponents (k, s). Specifically, we show that the flow is not C^2 near of the border lines of the well-posedness region obtained in [4]. Our result read as follows:

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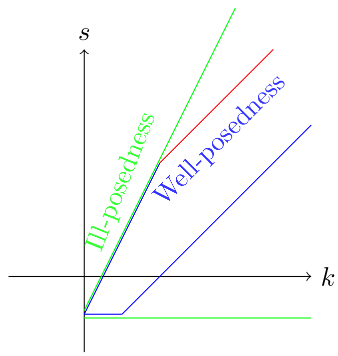
Theorem 3. *The Benney System (1.1) does not has flow C^2 in $H^k(\mathbb{R}) \times H^s(\mathbb{R})$ for any $\beta \in \mathbb{R}$ with k and s satisfying:*

$$s > 2k - \frac{1}{2} \quad or \quad s < -\frac{1}{2}.$$



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