Geometric interpretation of quasi modular forms

Hossein Movasati
IMPA

Quasi modular forms in the q-expansion form play a central role in mirror symmetry, in particular, they appear as generating functions counting the number of curves on K3 surfaces and the number of simply ramified covers of elliptic curves with marked points. On the other hand, in the framework of Algebraic Geometry modular forms, which are level zero quasi modular forms, are interpreted as sections of certain line bundles and this has arised many applications of modular forms in Number Theory and Arithmetic Algebraic Geometry. In these lectures I am going to discuss a generalization and applications of this picture for quasi modular forms. They turn out to be functions from the pairs of an elliptic curve and an element in its first de Rham cohomology to the base field.

They have functional properties with respect to the action ofan algebraic group.It turns out that the Ramanujan relations between Eisenstein series can be derived from the Gauss-Manin connection of families of elliptic curves.

Lecture 1: Algebraic de Rham cohomology of elliptic curves
Lecture 2: Gauss-Manin connection of families of elliptic curves
Lecture 3: Quasi-modular forms, Hecke operators, Ramanujan and Halphen differential equations
Lecture 4: Quasi modular forms arising in K3 surfaces and ramified coverings of elliptic curves.