ARBITRAGE IN OPTION TRADING: A BAYESIAN APPROACH FOR VERIFICATION

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Abstract. The concept of no arbitrage plays an essential role in the theories of finance. Under certain conditions, the *Fundamental Theorem of Asset Pricing* establishes a coherent and unique asset pricing framework in non-arbitraged markets, grounded on martingales processes. Accordingly, the analysis of the statistical distributions of financial assets can assist in understanding how participants behave in the markets, and may or may not engender conditions to arbitrage. Using a variance gamma stochastic model for prices, the study aims to verify, using the Bayesian test FBST, the difference between on the parameters estimated for the same financial asset obtained from two distinct markets. Specifically, the Bovespa Index distribution is investigated, with risk neutral parameters estimated based on options traded in (a) the Equities Segment and (b) the Derivatives Segment at the BM&FBovespa Exchange. Results seem to indicate significant statistical differences at some periods of time. To what extent this evidence is actually the expression of a perennial arbitrage between the markets is still an open question.

1. FIRST REMARKS

The non-occurrence of arbitrage is a concept that plays a key role in the theories of finance. Under no arbitrage there is a set of probability measures which defines, for states of nature, a coherent and unique pricing system for assets of financial markets. Specifically, it may assume the form of *stochastic discount factors*, with current prices expressed as functions of its future *payoff* structure, $S_i = E[\pi\delta_i]$, on which δ_i represents the stochastic payoff of the asset "i", S_i its price and E[ ] the mathematical expectation.[1]

The cornerstone for the no arbitrage pricing system rests on what in modern finance is known as the *Fundamental Theorem of Asset Pricing.* As pointed out by [7], preface, the theorem states that in a mathematical model of finance the no arbitrage principle holds if and only if there is an equivalent measure of probability that makes prices martingale processes. The martingale characterization of the processes is the essential condition for interpreting current prices of assets such as mathematical expectations of its future payoff structures. This relationship opened a vast field for the application of mathematics and stochastic integration in the

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[1]An informal definition of arbitrage is the possibility to obtain financial gains without incurring additional risks and net capital expenditures. For a formal definition see [7] chapter 2, 5 and 8.

theory of finance, whose driving force in recent decades has been to convert these principles into precise theorems.[2]

The objective of the study is to contextualize the arbitrage condition in terms of a statistical test hypothesis, using proper techniques for verification. More specifically, the concept of a *sharp* or *precise* hypothesis is necessary when evaluating the implications of the theorem with regard to the probability measures. Using the *Full Bayesian Significance Test*, FBST, the study proposes to comparatively evaluate the statistical distributions of the parameters that govern options trading in the Bovespa Index in two distinct markets, namely (a) the Equities Segment and (b) the Derivatives Segment at BM&FBovespa.[3] Detection of significant discrepancies may indicate distinct behaviors of participants in these markets that, in turn, may generate conditions to arbitrage.

The paper is organized as follows. Section 2 introduces the stochastic model. Section 3 provides a brief presentation of the relationship between the no arbitrage condition and risk neutral asset valuation.[4] Section 4 describes the proposed statistical test. Section 5 discusses the empirical data. Section 6 covers estimation procedures and displays the results. At the end final remarks are made.

2. MODELING THE STOCHASTIC PROCESS

Modern theory of option pricing is largely based on the work of [1] and [22]. Despite its importance, some of the assumptions made by them have been seriously questioned in literature, specifically the normality for continuously compounded returns. Strong empirical evidence, like volatility smiles and smirks or sporadical market crashes, suggested the need to extend the theory to more general statistical models, exhibiting skewness, kurtosis, and time-varying volatility structures. Some early examples of such extended models of particular historical importance were given by [25] and [18], for general overviews see [4] and [28]. Among these new models, one has gained considerable importance in the past few years, the variance gamma (VG) model, initially presented in [20] and [19], and generalized in [18]. According to the authors, the VG model correctly adjusts the volatility smile, because it incorporates a parameter related to kurtosis. In contrast, the prize for asymmetry is treated with the insertion in the model of parameters associated with skewness. Tests presented in these literature show that pricing errors related to these caveats tend to be minimized.

Specifically the VG model is an extension of an Ito process, with constant drift and diffusion, with time behaving as a random variable determined by a stochastic gamma distribution. The idea here is that time accounts for relevant economic action, having as many random jumps as the level of market activity generates. With a few small changes in the terminology of the authors,

$$X(t;\sigma,\nu,\mu) = \mu \cdot \gamma(t;1,\nu) + \sigma \cdot B(\gamma(t;1,\nu)), \tag{2.1}$$

[2]The original pillars of the Fundamental Theorem of Asset Pricing were outlined in [12] and [13]. [8] provides a detailed construction of the reasoning. For a more general mathematical approach see [7].

[3]BM&FBovespa is an multi-asset exchange house located in Brazil. In terms of market capitalization the exchange can be considered the 3rd largest in the world.

[4]For further arguments see [13] and [8] chapters 5 and 6.

in which $X(t;\sigma,\nu,\mu)$ is the VG process, μ is the drift, σ is the volatility, *B(t)* is a standard Brownian Motion and $\gamma(t;1;\nu)$ is a gamma process with unit mean rate and ν variance rate.

One appealing characteristic of the VG model is that it incorporates the Ito Process and the Black and Scholes model as a parametric special case. In particular, conditional on the realization of a random time change g, the process $X(t;\sigma,\nu,\mu)$ is normally distributed. Thus the unconditional density for X process can be obtained integrating out g, based on a gamma distribution

$$f(X) = \int_0^\infty \frac{1}{\sigma\sqrt{2\pi g}} \exp\left(\frac{-(X-\mu g)^2}{2\sigma^2 g}\right) \frac{g^{\frac{t}{\nu}-1}\exp\left(\frac{-g}{\nu}\right)}{\nu^{\frac{t}{\nu}}\Gamma\left(\frac{t}{\nu}\right)} dg. \tag{2.2}$$

From VG stochastic motion it is possible to determine the behavior of prices, here expressed by

$$S(t) = S(0)\cdot\exp(r\cdot t + X(t;\sigma,\nu,\mu) + \omega\cdot t), \tag{2.3}$$

in which $\omega = \nu^{-1}\cdot\ln(1-\mu\cdot\nu-\sigma^2\cdot\nu\cdot 2^{-1})$, is defined so that $E[S(t)] = S(0)\cdot\exp(r\cdot t)$, with r the no-risk interest rate. Under this setting of the stochastic process it can be seen that $\ln(E[S(t)]/S(0)) = r\cdot t$, implying that a risk neutral probability measure is considered. Moreover, it is clear that under this measure the present value of prices is a martingale process, $E[S(t)\cdot\exp(-r\cdot t)] = S(0)$.

Based on this characterization, it is possible to extend the concept and set the price of a call option of an asset that follows a VG process, with strike value of K, maturing at t. In a risk neutral world, it can be expressed as the martingale condition $c(S(0);K,t) = \exp(-r\cdot t)\cdot E[max[S(t)-K;0]]$. [18] demonstrate that this mathematical expectation can be analytically expressed by

$$\begin{aligned} c(S(0);K,t) = {} & S(0)\cdot\Psi\left(d\cdot\sqrt{\frac{1-c_1}{\nu}},(\alpha+s)\cdot\sqrt{\frac{\nu}{1-c_1}},\frac{t}{\nu}\right) \\ & -K\cdot exp(-r\cdot t)\cdot\Psi\left(d\cdot\sqrt{\tfrac{1-c_2}{\nu}},(\alpha\cdot s)\cdot\sqrt{\tfrac{\nu}{1-c_2}},\tfrac{t}{\nu}\right), \end{aligned} \tag{2.4}$$

with

$$d = \frac{1}{s}\left[ln\left(\frac{S(0)}{K}\right) + r\cdot t + \frac{t}{\nu}\cdot ln\left(\frac{1-c_1}{1-c_2}\right)\right], \tag{2.5}$$

$$\alpha = \zeta\cdot s, \zeta = \frac{-\mu}{\sigma^2}, s = \frac{\sigma}{\sqrt{1+\left(\frac{\mu}{\sigma}\right)^2\cdot\frac{\nu}{2}}}, \tag{2.6}$$

$$c_1 = \frac{\nu\cdot(\alpha+s)^2}{2}, c_2 = \frac{\nu\cdot\alpha^2}{2}. \tag{2.7}$$

Ψ is a function defined in terms of Second Order Modified Bessel Functions and degenerate hypergeometric functions of two variables, as shown in the Appendix of the work of the authors.

3. FUNDAMENTAL THEOREM AND ARBITRAGE

From an economic standpoint, with the exception of imperfect markets, the assumption of no arbitrage appears as a natural condition of the functioning of the markets and behavior of economic agents. As previously stated, *Fundamental Theorem of Asset Pricing* formalizes the relation between arbitrage and the stochastic behavior of assets. Accordingly, the no arbitrage condition on the markets entails a martingale property of stochastic processes. A consequence of this implication is the existence of a coherent and unique system for asset pricing, sometimes referred to as risk neutral valuation.

Under this particular probability measure, the present value of all assets must equal its expected future value, or in a similar manner, must equal its value discounted at the riskless rate, $E[S(t) \cdot \exp(-r \cdot t)] = S(0)$. A consequence of the approach is that all assets trading in this economy must have the same expected rate of return, given by the risk-free interest rate. The fact that the approach does not need any additional hypothesis, but the no arbitrage condition, releases it from imposing any restriction on financial agents' preferences. On this manner, risk averse, risk neutral, and pro risk agents might value the asset the same way.

The same reasoning can be extended from assets to derivatives based on these assets using martingale representation property. The pricing formula, presented previously for the VG approach, is a consequence of this property, with the expectation calculated over S following a variance gamma stochastic process. Thus, the close relationship is evident among the statistical distribution of asset prices, the formulations of pricing equations, and the opportunity to profit without risk–the principle of arbitrage.

4. FBST: A TEST PROPOSAL

A simple manner of evaluating the phenomenon of arbitrage is to study the parameters of the statistical distributions that govern the behavior of asset prices. In particular, under the VG process the no arbitrage condition can be expressed in terms of a test hypothesis,

$$H: \quad [\sigma_a, \nu_a, \mu_a] = [\sigma_b, \nu_b, \mu_b] \ .$$

If the condition holds true, the parameters on (a) the Equities Segment and (b) the Derivatives Segment should be the same, given that payoff structure, or contingent claim, is the same for option trading on both of them. A statistically significant difference among them may be indicative of an arbitrage opportunity.

The hypothesis implies reducing by half the dimension (or degrees of freedom) of the parameter space under consideration. Hence, the abstract no arbitrage condition is translated into a concrete, sharp, statistical hypothesis, meaning that the dimension of the null hypothesis is strictly smaller than that of the parameter space, $dim(\Theta_0) < dim(\Theta)$. Although direct, treatment and measurement of the statistical support of sharp hypotheses pose a number of technical and epistemological difficulties. Not only the traditional Bayesian approach, which seeks to answer questions related to hypothesis testing using the Bayes Factor and the use of conceptual procedures of decision theory, but also the classical frequentist statistician, face difficulties in dealing with the precise, or sharp, hypotheses.[5]

The shift on the way of evaluating test hypotheses observed in the literature, from the calculation of a relative measure regarding the probabilities of the hypotheses towards measuring the plausibility of the test hypothesis in relation to the other parameter values, has emerged as a successful alternative in the treatment of precise hypotheses. Among the Bayesian approaches, other than those directed to the Bayes Factor and decision theory, is FBST, *Full Bayesian Significance Test.* The

[5]For an interesting discussion on precise hypothesis see [27].

test was initially proposed by [23] as a way of evaluating precise hypotheses by calculating measures of evidence. The test is methodologically considered *Full Bayesian*, being also consistent with the principle of Bayesian Likelihood. For some other characteristics of FBST see [26] and [21].

The characterization of FBST can be done segregating it in two parts, the first set by determining a region tangent to the test hypothesis, called the Highest Posterior Density Set (HPDS), and second by calculating the credibility of the HPDS. Although general procedures, the test was especially designed to evaluate precise hypotheses. For the first part, consider the precise test hypothesis $H : \theta \in \Theta_0$, in which $\Theta_0 \subset \Theta \subseteq R^n$ and Θ is the parameter space. Let θ^* be the maximum argument of the *posterior* distribution $h(\theta \mid x)$ under the test hypothesis, $\theta^* = argmax_{\theta \in \Theta_0} h(\theta \mid x)$. Thus, the interval HPDS can be defined as

$$HPDS = \{\theta \in \Theta \mid h(\theta \mid x) > h(\theta^* \mid x)\}. \tag{4.1}$$

The range includes all values of the parameter vector θ for which the *posterior* density assumes values greater than the highest point within the range defined by the test hypothesis.

To calculate the credibility of the whole HPDS, it is necessary to integrate the *posterior* density in the interval, mathematically expressed by

$$k = \int_{\theta \in HPDS} h(\theta \mid x) d\theta. \tag{4.2}$$

The complement of the probability of the HPDS, also known as ***e-value***, is a measure of the statistical evidence of the test hypothesis proposed by [23],

$$Ev(H) = 1 - k. \tag{4.3}$$

As a measure of probability of HPDS, the evidence in favor of the test hypothesis varies in the interval [0, 1]. H is considered as more plausible the closer the value of the evidence is to unity. The logic behind the evidence measurement is that if the credibility of HPDS is relatively high, then the set of values of the parameters belonging to Θ_0 occupy a low probability region in the *posterior*.

5. MARKET INFORMATION

As previously stated, in order to verify the test hypothesis H it was selected two separate trading environments but that ultimately operate the same underlying asset, Bovespa Index, also known as Ibovespa. In particular, the study covers options on Ibovespa spot, traded at (a) Equities Segment of the BM&FBovespa Exchange and options on Ibovespa Futures at (b) Derivatives Segment operated in the same institution. Although there is a distinction of the contract underlying asset in each kind of option, when considering the process of settlement at maturity of the positions both markets depend on the same economic variable, namely the value of Ibovespa Settlement.[6]

At maturity, the options on the Equities Segment financially settle accordingly to the difference between the Ibovespa Settlement and the strike of the option. In the case of options on Ibovespa Futures at Derivatives Segment on the expiration date the parties take positions in the Future Index contract, starting to enforce the

[6] The Ibovespa Settlement is the arithmetic mean of the Bovespa Index in the three last hours of trading, including the end of the closing call. This mean is taken on the last trading day, as defined by the BM&FBovespa.

rules of this market. However, on the expiration date these futures also liquidate financially accordingly to the Ibovespa Settlement. Thus, by promoting exercise on the expiration date, the parties are implicitly assuming the settlement based on the difference between the Ibovespa Settlement and the strike of the option, this last variable considered as the traded value for the rules of the Index Futures market. Besides this subtle difference, most of the other important aspects of the contracts as maturity, style and so on are the same for both segments.

For purposes of statistical analysis, the study worked with the business information of call options on the Ibovespa for two expiring dates, Feb./2012 and Apr./2012. Because trading in Ibovespa Future option at Derivatives Segment only become bulky two months before its maturity, the periods considered for each expiring date were 12/15/2011 to 02/14/2012 and 02/22/2012 to 04/15/2012 respectively. The informations were captured for Equities Segment and Derivatives Segment for strikes ranging from 54,000 to 72,000, with bands at 1,000 points. In order to avoid non-synchronization of the data it was sought to capture trades close to the marking to market call. The Ibovespa spot values were obtained near the time of each trade. In total the study worked with 840 observations.

As a proxy of the value of the Reference Interest Rate of the economy it was used the fixed rate implicit in DI Futures contracts traded at BM&FBovespa. In the case that the maturity of the DI Futures differed from maturities of the options, it was applied the exponential interpolation based on 252 business day convention to estimate the interest rates of the options.[7]

6. ESTIMATION AND RESULTS

A standard approach to perform Bayesian analysis in finance econometrics is to formulate an empirical observation error model, combine it with the basic stochastic models driving the price evolution of financial assets and derive a joint empirical likelihood function, see [5], [16], [14] and [15].

Following the approach of [9] and [18] a simple observation error model is formulated for observed prices, w_i, relative to theoretical model prices, $\hat{w}_i$, using an exponential multiplicative structure, $w_i = \hat{w}_i \exp(\eta\epsilon_i - \eta^2/2)$, where ϵ_i stands for the standard white noise, that is, a zero-mean unit-variance Gaussian process. Thereby, $w_i \sim \ln N(\ln \hat{w}_i - \eta^2/2, \eta^2)$. According to [18] this formulation is well suited to deal with heteroskedasticity in option prices for different strikes.

An additional hypothesis is that there is no dependence structure for errors within each one of the segments considered, (a) Equities and (b) Derivatives, and across them, summarizing the likelihood function to

$$\ell(\eta, \sigma_a, \nu_a, \mu_a, \sigma_b, \nu_b, \mu_b | w_a, w_b) = \ell(\eta, \sigma_a, \nu_a, \mu_a | w_a) \cdot \ell(\eta, \sigma_b, \nu_b, \mu_b | w_b) \tag{6.1}$$

with,

$$\ell(\eta, \sigma, \nu, \mu | w) = \prod_{i=1}^{M} \frac{1}{w_i\sqrt{2\pi\eta^2}} \cdot \exp\left(-\frac{\left(\ln(w_i) - \ln(\bar{c}(\sigma, \nu, \mu; W)) + \eta^2/2\right)^2}{2\eta^2} \right), \tag{6.2}$$

in which W represents all the economic variables of Eq. 4, $w = \{w_1, w_2, ..., w_M\}$ and $\bar{c}(\sigma, \nu, \mu; W) = c(S; K, t) = \hat{w}_i$. To obtain the Bayesian *posterior* density, a

[7]The source of the data on the options, DI Futures and Ibovespa spot was BM&FBovespa.

Table 1. Maximum Likelihood Estimators of VG Model Risk Neutral Parameters. The most liquid week for Feb./2012 maturity (5^{th} week) and for Apr./2012 maturity (14^{th} week). The first row for each week exhibits the parameters under unconstrained *posterior* and second row the parameters under null hypothesis H.

Week	*#Observations*	η	σ_a	ν_a	μ_a	σ_b	ν_b	μ_b
05	96	0.3208	0.1723	0.0003	0.0008	0.1849	0.0003	0.0010
		0.3208	0.1748	0.0003	0.0008	0.1748	0.0003	0.0008
14	70	0.2957	0.2339	0.0066	-0.0793	0.1939	0.0012	0.0054
		0.2957	0.4556	0.0169	0.0738	0.4556	0.0169	0.0738

non-informative *prior* of the form $h(\eta) \propto 1/\eta$ was used, updating the likelihood accordingly with

$$h(\eta, \sigma_a, \nu_a, \mu_a, \sigma_b, \nu_b, \mu_b | w_a, w_b) \propto h(\eta) \cdot \ell(\eta, \sigma_a, \nu_a, \mu_a, \sigma_b, \nu_b, \mu_b | w_a, w_b). \tag{6.3}$$

In order to verify the no arbitrage hypothesis, H, the FBST is implemented using the aforementioned *posterior*. The procedure presents a statistical measure of the similarity of estimated parameters for both segments considered, the ***e-value***. The test is performed on a weekly basis, as initially proposed by [18]. For the Feb./2012 maturity, 10 evaluations of the test are applied, ranging from the 51^{st} week of 2011 to 7^{th} week of 2012. For the Apr./2012 maturity, the test is performed from 8^{th} week of 2012 to 15^{th} week of 2012. As can be observed, there is no overlapping of data use on the tests, meaning that only one maturity is evaluated each time and weeks are treated independently.

As previously stated, FBST is performed in two steps, first by optimization and then by integration. For the present case, but also in general use of the test, the optimization and integration steps are performed numerically. The optimization step can be implemented using general purpose numerical optimization algorithms, like [2] and [3]. The integration step is tailor coded for the specific application using standard computational tools and techniques of Bayesian statistics like Monte Carlo and Markov Chain Monte Carlo procedures, see [11] for a general overview and [10] for surveys of MCMC algorithms with application in Bayesian statistics.

Using the values of the trades observed in the market, the maximum likelihood method can be applied to estimate the desired parameters, given by the volatility σ, the parameter associated with kurtosis ν, and the parameter related to the asymmetry or skewness μ. Table 1 presents the results of the estimated parameters for the most liquid weeks: for Feb./2012 the 05^{th} week, and for Apr./2012 maturity, the 14^{th}. Specifically two sets of parameters are displayed for each week, the ones estimated under the hypothesis H, $\sigma = \sigma_a = \sigma_b$, $\nu = \nu_a = \nu_b$, $\mu = \mu_a = \mu_b$, second row, and also under the unconstrained version of the *posterior* density, $(\eta, \sigma_a, \nu_a, \mu_a, \sigma_b, \nu_b, \mu_b)$, first row.

While the optimization step permits the establishment of estimated parameters, and consequently the HPDS, it will be through the integration process that the probability measure of the set will be determined. As with most Bayesian inferential procedures, integration plays an important role in FBST. Nevertheless, the difficulty of obtaining an analytical formulation for the integrals of the *posterior* distribution $h(\eta, \sigma_a, \nu_a, \mu_a, \sigma_b, \nu_b, \mu_b | w_a, w_b)$, derived from the likelihood function characterized

TABLE 2. *Full Bayesian Significance Test*: Feb./2012 maturity, weekly data from 12/15/2011 to 02/14/2012. The ***e-value*** of the no arbitrage hypothesis, H. Values close to the unit mean that there is statistical evidence in favor of the hypothesis.

Week	51	52	53	01	02	03	04	05	06	07
e-value	0.01	0.85	1.00	0.98	0.64	0.97	0.96	1.00	0.71	1.00
# Observations	51	83	21	60	38	69	49	96	34	11

above and the non-informative *a priori*, engendered the need to resort to techniques of integration via Monte Carlo to determine the ***e-value***. At a higher level of an MCMC process, Gibbs Sampling and Metropolis-Hastings techniques are combined to simulate the random sample of the parameters for the numerical integration.[8]

The Gibbs technique permits the simulation of the joint distribution resorting to the conditional values of *posterior*. In particular, using a cyclical chain along the conditional densities, the parameters could be obtained, following through

$$\sigma_a^{(k+1)} \sim h(\sigma_a | w_a, w_b, \eta, \nu_a^{(k)}, \mu_a^{(k)}, \sigma_b^{(k)}, \nu_b^{(k)}, \mu_b^{(k)}), \tag{6.4}$$

$$\nu_a^{(k+1)} \sim h(\nu_a | w_a, w_b, \eta, \sigma_a^{(k+1)}, \mu_a^{(k)}, \sigma_b^{(k)}, \nu_b^{(k)}, \mu_b^{(k)}), \tag{6.5}$$

...

$$\mu_b^{(k+1)} \sim h(\mu_b | w_a, w_b, \eta, \sigma_a^{(k+1)}, \nu_a^{(k+1)}, \mu_a^{(k+1)}, \sigma_b^{(k+1)}, \nu_b^{(k+1)}). \tag{6.6}$$

Even though the conditioning diminished the complexity of the task, simulating from any one of the *posterior* conditionals is still a problem. Although univariate, the statistical distributions are non-standard. To handle this drawback, Metropolis-Hastings was implemented. Specifically, a Gaussian kernel, $N(0, \xi)$, was employed to dynamically generate proposals, controlling the acceptance/rejection rate through adjustments to its variance. In order to derive the random sample from the combinations of the techniques, the burn-in period, the spacing among realizations and the size of the sample were respectively set to 200, 10, and 500. Fine tuning of these parameters for the very best performance in FBST applications can be accomplished using the error control method presented in [17].

Table 2 and Table 3 present the results of the implementation of FBST for the two maturities respectively considered. For Feb./2012 maturity, with the exception of the 51^{st} week, the computed ***e-values*** indicate evidence for the equivalence between the parameter estimates for the Equities and Derivatives Segments. Conversely, for Apr./2012 maturity, the empirical observation seems to show statistical divergence of parameters in more than one verification. In these specific weeks, low values of FBST denote that the respective H hypotheses are located at inferior levels of the *posterior* density. In the light of these results, evidence appears to exist that at some times there may be divergence of participants' behavior when determining the options prices in each segment. As modeled here, this divergence expresses itself on the difference of parameters of the price movements for each segment.

[8]For a direct discussion of Metropolis-Hastings see [6].

TABLE 3. *Full Bayesian Significance Test*: Apr./2012 maturity, weekly data from 02/22/2012 to 04/15/2012. The ***e-value*** of the no arbitrage hypothesis, H. Values close to the unit mean that there is statistical evidence in favor of the hypothesis.

Week	08	09	10	11	12	13	14	15
e-value	0.78	0.98	0.01	1.00	0.00	0.00	0.00	0.37
# Observations	14	41	44	24	37	42	70	25

7. FINAL REMARKS

In the preceding sections a coherent framework was proposed for translating the no arbitrage condition in financial markets into a sharp test hypothesis, followed by the determination of a test procedure for verification. Generally, the framework was based on:

(1) the *Fundamental Theorem of Asset Pricing*;
(2) the variance gamma option pricing model;
(3) a carefully defined empirical likelihood function well suited for data analysis in financial econometrics;
(4) the *Full Bayesian Significance Test* methodology;
(5) the efficient implementation of computational algorithms; and
(6) a carefully assembled data bank with price series of options in the Bovespa Index traded at (a) the Equities Segment and (b) the Derivatives Segment of the BM&FBovespa exchange.

The empirical estimation exhibits that under the testing approach considered, at some moments, there appear to be indicators of divergence between the parameterization of the function for pricing call options between the Equities and Derivatives Segments. These findings suggest that during these specific periods there can be conditions to arbitrage. To what extent the evidence is actually the expression of a perennial arbitrage opportunity between the markets is still an open question. What is a fact is that at some points in time traders appear to price the same future payoff structure differently. It is also certain that the Bovespa Index trend and volatility pattern changed significantly during the period of investigation. How many of the aforementioned findings are influenced by these movements also deserves careful analysis.

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[9]The ideas presented here do not express any private or public institution view and are solely related to the authors' personal opinion.