A Level Set Approach to Optimal Stopping Time in American Options

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Poster Section - RiO 2011

- We analyze the inverse problem related to the determination of the *stopping time* in an American put option price model in a finite horizon.

- Given a set of American put options we propose a level set regularization method to obtain a stable approximated solution that identifies the continuation set C and the stopping set D.

Model

The model is given by the **free boundary problem** with unknown boundary $b = b(t)$ satisfying

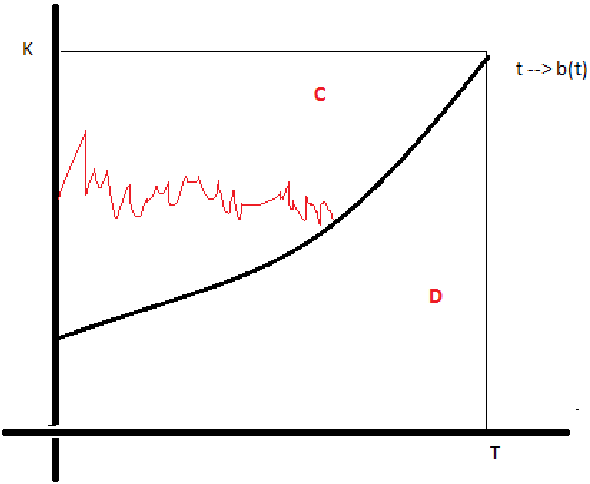
$$\begin{cases} V_t + rxV_x + \frac{\sigma^2}{2}x^2 V_{xx} = rV & \text{in} \quad C \\ V(t,x) = (K-x)^+ & \text{for} \quad x = b(t) \\ V_x(t,x) = -1 & \text{for} \quad x = b(t) \\ V(t,x) > (K-x)^+ & \text{in} \quad C \\ V(t,x) = (K-x)^+ & \text{in} \quad D \end{cases} \tag{1}$$

where $V(t,x)$ is the arbitrage free price of an American Option.
Here

$$C := \{(t,x) \in [0,T] \times (0,\infty) \, : \, x > b(t)\} \tag{2}$$

is the continuation set
and the stopping set $\overline{D}$ is the closure of the

$$D := \{(t,x) \in [0,T] \times (0,\infty) \, : \, x < b(t)\} \, . \tag{3}$$


K
t --> b(t)
C
D
T

The inverse problem

Given a set of American put option values $\{V_j\}_{j=1}^n$, we want:

- to identify the optimal stopping boundary $b(t)$.
- to identify the continuation set C and the stopping set $\overline{D}$.

This process can be formally described by the operator equation

$$F(b(t)) = V, \quad (4)$$

where $F : D(F) \subset L^1(\Omega) \to L^2(\Omega)$.