UNIQUENESS OF QUASI-EINSTEIN METRICS ON 3-DIMENSIONAL HOMOGENEOUS RIEMANNIAN MANIFOLD

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joint work with A. Barros and J.F. Silva

One of the motivation to study m-quasi-Einstein metrics on a Riemannian manifold (M^n, g) is its closed relation with warped product Einstein metrics, see e.g. [3]. For instance, when m is a positive integer, m-quasi-Einstein metrics correspond to exactly those n-dimensional manifolds which are the base of an $(n+m)$-dimensional Einstein warped product. It is important to detach that gradient 1-quasi-Einstein metrics satisfying $\Delta e^{-f} + \lambda e^{-f} = 0$ are more commonly called *static metrics* with cosmological constant λ. These static metrics have been studied extensively because their connection with scalar curvature, the positive mass theorem and general relativity, for more details see e.g. [1] and [4].

The study of 3-dimensional homogeneous Riemannian manifolds is done, in general, according to the dimension of its isometry group $Iso(M^3, g)$, which can be 3, 4 or 6. Following this trend we present here a complete description of m-quasi-Einstein metrics, when this manifold compact or not compact provided $dim\, Iso(M^3, g) = 4$. In addition, we shall show the absence of such structure on Sol^3, which corresponds to $dim\, Iso(M^3, g) = 3$. When $dim\, Iso(M^3, g) = 6$ it is well known that M^3 is a space form. In this case, its canonical structure gives a trivial example. In particular, we shall prove that Berger's spheres carry naturally a non trivial structure of quasi-Einstein metrics. Since they have constant scalar curvature, their associated vector fields can not be gradient, this shows that Perelman's Theorem can not be extend to quasi-Einstein metrics. Moreover, these examples show that Theorem 4.6 of [5] can not be extended for a non gradient vector field. Finally, we prove that if (M^3, g, X, λ) is a non compact 3-dimensional homogeneous Riemannian manifold such that g is a m-quasi-Einstein metric, then, either M^3 is a space form or M^3 is $E^3(\kappa, \tau)$ such as our Example obtained in this work.

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