

Finiteness of algebraically primitive closed $SL(2, \mathbb{R})$ -orbits in moduli spaces

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Abstract:

An interesting (mostly open) question about the action of $SL(2, \mathbb{R})$ on moduli spaces of Abelian differentials (translation surfaces) of genus $g > 1$ concerns the classification of ergodic $SL(2, \mathbb{R})$ -invariant probability measures.

After the works of K. Calta and C. McMullen in 2004, the complete list of $SL(2, \mathbb{R})$ -invariant probability measures in genus $g=2$ is known. More recently, A. Eskin and M. Mirzakhani showed the "Ratner-type" result that all ergodic $SL(2, \mathbb{R})$ -invariant probability measures are affine (i.e., "close to algebraic"). This profound theorem gives hope for a classification result in all genera, but it should be taken with a grain of salt because the more basic problem of classifying all closed $SL(2, \mathbb{R})$ -orbits is not available.

In this direction, using algebro-geometrical methods, M. Bainbridge and M. Moller showed the finiteness of closed $SL(2, \mathbb{R})$ -orbits satisfying a condition called "algebraically primitivity" in several particular cases. In this talk, we will use a dynamical approach (based on the key notion of Hodge-Teichmüller planes and some computations of Zariski closure of certain monodromy representations) to show the finiteness of algebraically primitive closed $SL(2, \mathbb{R})$ -orbits in some cases not covered by M. Bainbridge and M. Moller. This is a joint work with Alex Wright.