Dynamical Markov and Lagrange Spectra and Geodesic Flows

Carlos Gustavo Moreira - IMPA

Let φ be a diffeomorphism of a surface M and Λ a horseshoe associated to φ. Given a continuous function $f\colon M \to \mathbb{R}$, we define the *dynamical Lagrange spectrum* by

$$L(f,\Lambda) = \left\{ \limsup_{n\to\infty} f(\varphi^n(x)) : x \in \Lambda \right\},$$

and the *dynamical Markov spectrum* by

$$M(f,\Lambda) = \left\{ \sup_{n\in\mathbb{Z}} f(\varphi^n(x)) : x \in \Lambda \right\}.$$

We prove that typically, if the Hausdorff dimension of Λ is larger than 1, then for a (large) set of "typical" functions $f \in C^1(M,\mathbb{R})$, their dynamical Markov and Lagrange spectra have persistently non-empty interior.

Let M be a manifold, and let X be a complete vector field over SM. We define the dynamical Lagrange and Markov spectra associated to the pair (f,X), where $f \in C^0(M,\mathbb{R})$ by

$$L(f,X) = \left\{ \limsup_{t\to\infty} f(X^t(x)) : x \in M \right\}$$

and

$$M(f,X) = \left\{ \sup_{t\in\mathbb{R}} f(X^t(x)) : x \in M \right\}$$

respectively, where $X^t(x)$ is the trajectory of the flow of X beginning in x.

We prove that if M is a surface with a Riemannian metric g with negative curvature, bounded between two negative constants, and finite volume, and SM is the unit tangent bundle of M then there is a metric g_0 which may be taken arbitrarily close to the original metric g, such that the Markov and Lagrange dynamical spectra associated to any field close enough to the geodesic field in SM of the metric g_0 has persistently non-empty interior for an open and dense set of functions in $C^2(SM,\mathbb{R})$.

This is a joint work with Sergio Romaña Ibarra.