

The Time of Bootstrap Percolation

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Bootstrap percolation, a simple cellular automaton introduced by Chalupa, Leath and Reich in 1979, can be viewed to model the spread of an infection. Let G be a graph, and for every vertex x , let $r(x)$ be a natural number. Starting with a set A_0 of ‘infected’ vertices of G at time 0, in bootstrap percolation with threshold function r the infection spreads according to the following deterministic local update rule: a vertex x with at least $r(x)$ infected neighbours at time t becomes infected at time $t + 1$, and every infected vertex remains infected for ever. The set A *percolates* if eventually every vertex of G is infected; the first time when every vertex is infected is the *time of percolation*.

In the past three decades, much work has been done on bootstrap percolation on finite grids of a given dimension in which the initial set A is obtained by selecting its vertices at random, with the same probability p , independently of all other choices. The focus has been on the *critical probability*, the value of p at which the probability of percolation is $1/2$. The first half of my talk will be a review of some of the fundamental results concerning critical probabilities obtained by Aizenman, Lebowitz, Schonman, Cerf, Cirillo, Manzo, Holroyd and others, and by Balogh, Morris, Duminil-Copin and myself. The second half will be about the very recent results I have obtained with Holmgren, Smith and Uzzell on the time a random initial set takes to percolate.