The Phase Transition of Random Graphs

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Over fifty years ago, Erdős and Rényi proved the striking result that the structure of $G(n, m)$, the random graph with n vertices, and m edges selected uniformly at random, undergoes a phase transition (as m increases) around the point $m = n/2$. For the past quarter of a century, this phase transition has been studied in great detail, with a host of detailed results emerging, such as the size of the scaling window, and the behaviour of the giant component outside this window. In particular, Pittel and Wormald proved a deep and difficult theorem about the limiting distribution of the size of the giant component above the scaling window of the phase transition. Later, Nachmias and Peres used martingale arguments to study Karp's exploration process, and obtained a simple proof of a weak form of this result. Even more recently, Riordan and I have found a simple proof of the full result of Pittel and Wormald.

In this lecture I shall review some of the early theorems concerning the giant component, together with some of the much finer results that have been obtained in recent years. I shall also sketch the simple proof mentioned above.