The wind-tree model

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IMJ, Paris VII

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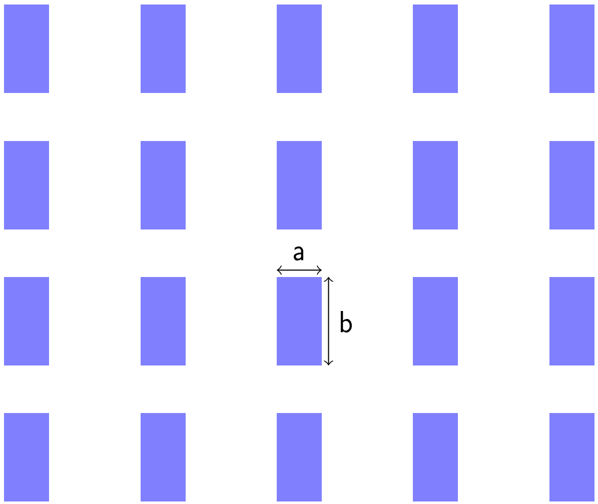

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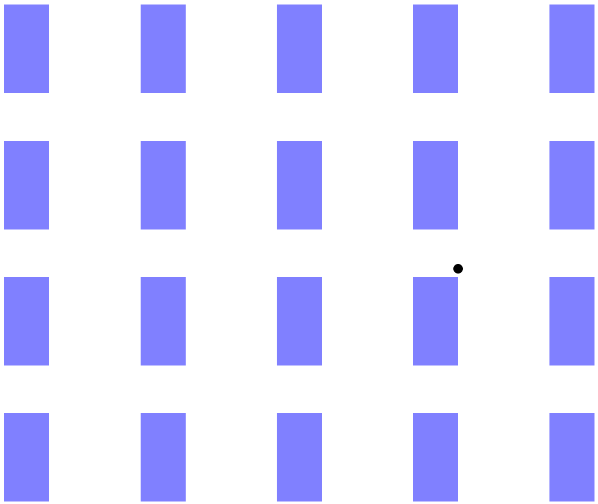

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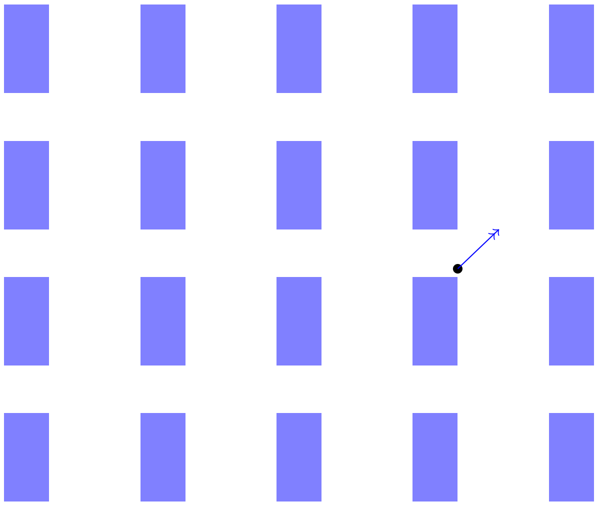

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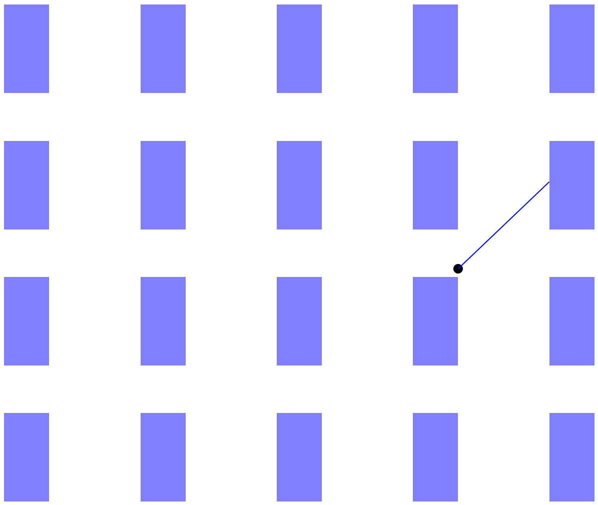

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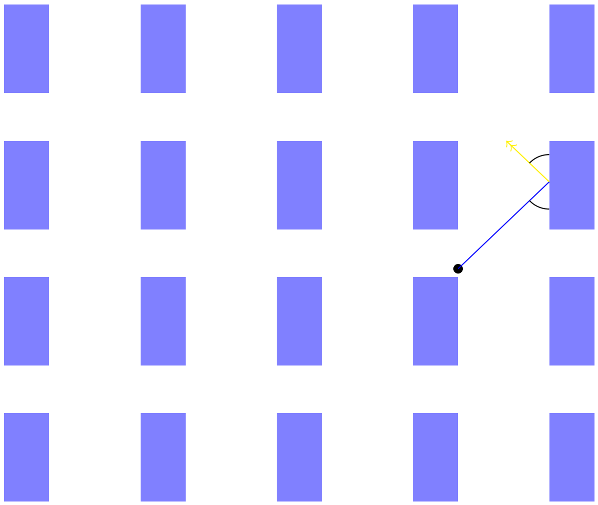

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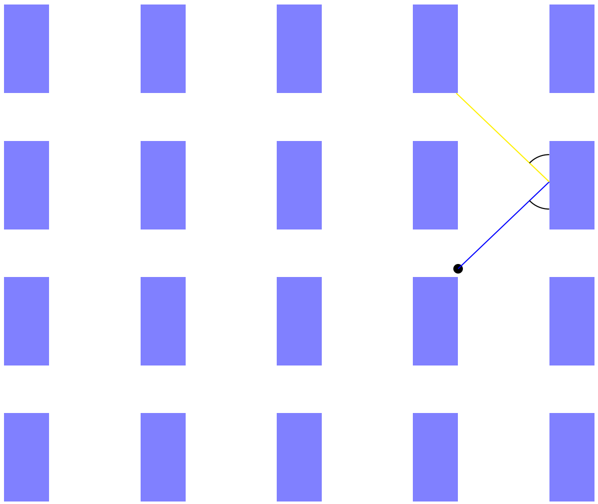

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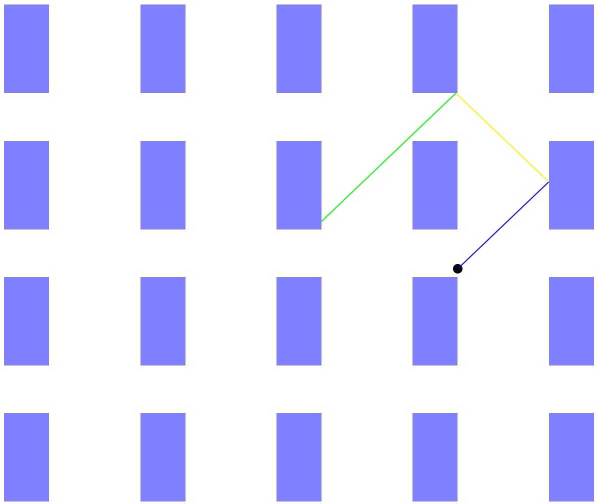

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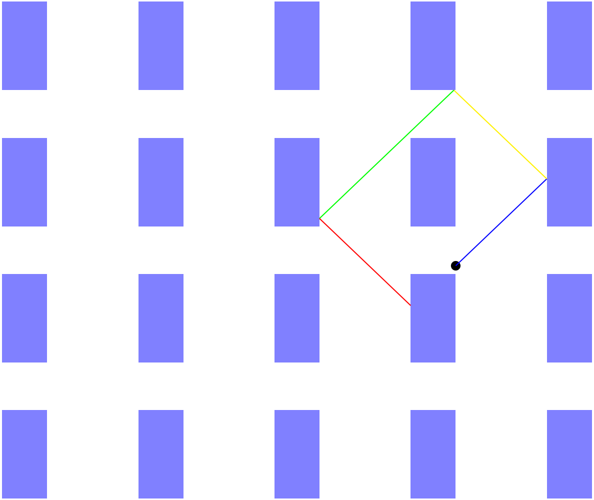

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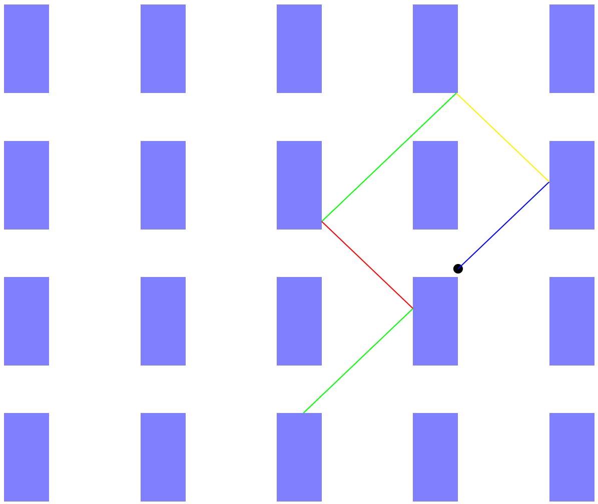

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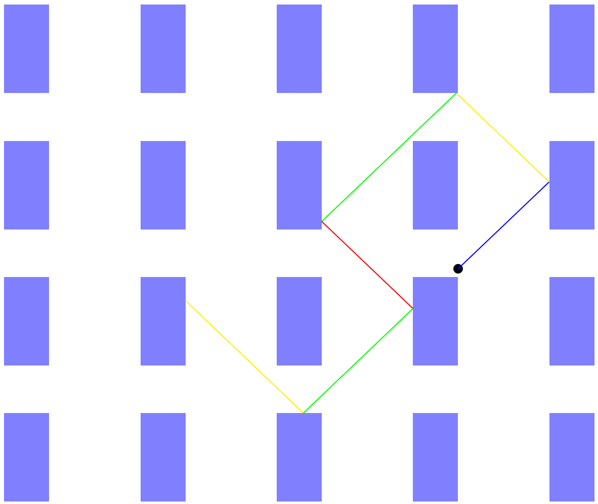

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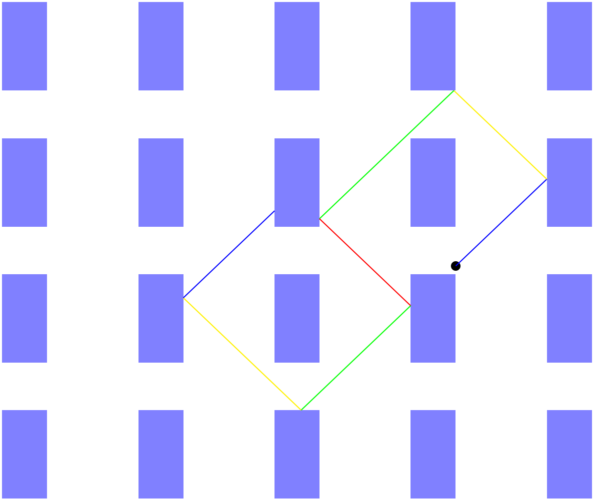

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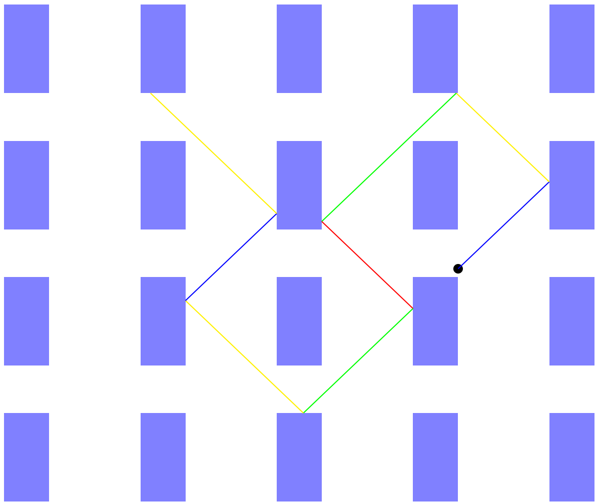

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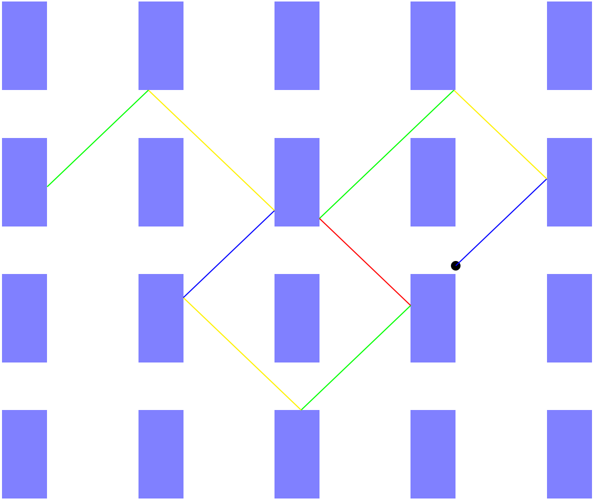

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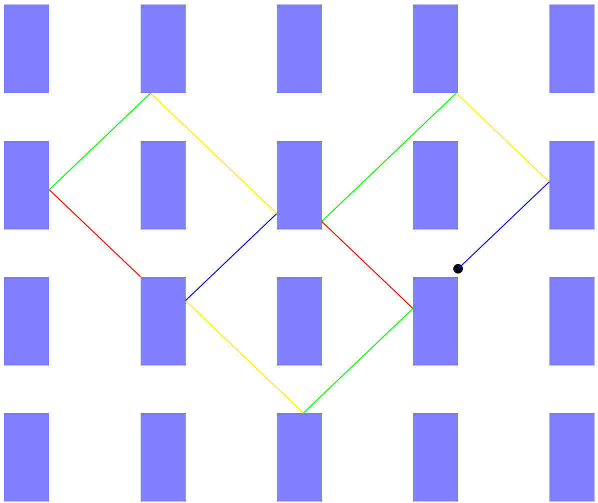

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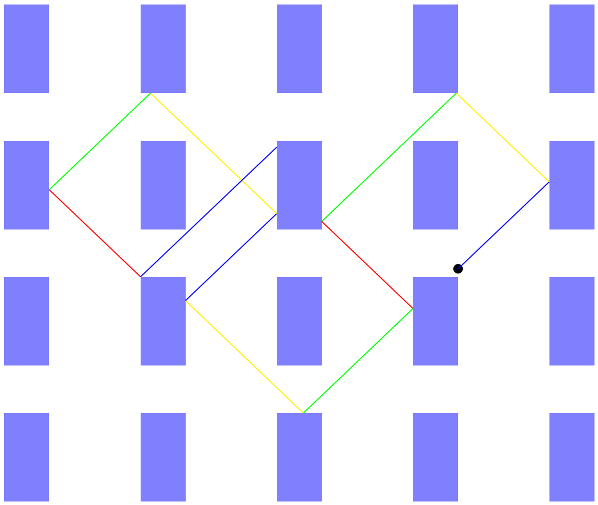

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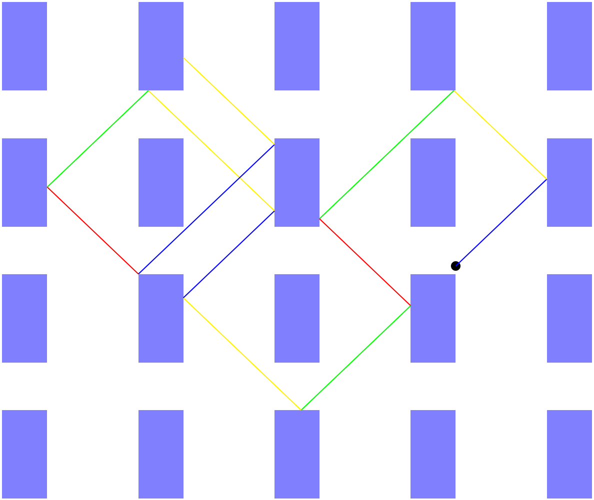

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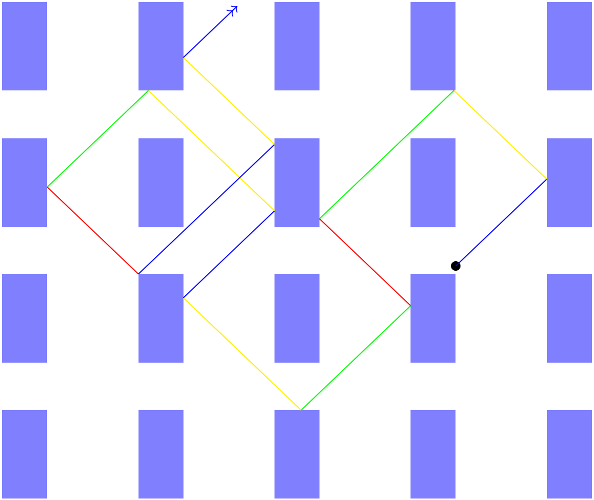

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2. *For many rational parameters a, b, the set of directions θ for which the diffusion is subpolynomial (ie $\operatorname{dist}(x, \phi_t^{a,b,\theta}) = O(t^\epsilon)$ for any $\epsilon > 0$) has Hausdorff dimension greater than $1/2$*

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Some questions

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5. random (but still rectangular and aligned) scatterers ?