Geometric properties of partially hyperbolic attractors

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Context

$f : M \to M$ a C^1-diffeomorphism. Λ is a partially hyperbolic set ($T_\Lambda M = E^s \oplus E^c \oplus E^u$) saturated by W^u.

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- Λ is a *quasi-attractor*: Decreasing intersection of attractors which is chain-recurrent (for example, a transitive attractor)
- Λ is a Hausdorff limit of quasi-attractors Λ_n.

Goal

- Study the geometry of minimal W^u-saturated sets.
- Finiteness of such minimal sets.

Motivations

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Bifurcations (such as tangencies) should be the obstructions for well understood phenomena (hyperbolicity/ finite spectral decomposition).

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Conjecture (Bonatti-Tameness Conjecture)

A C^1-generic diffeomorphism far from homoclinic tangencies has finitely many basic pieces ("spectral decomposition").

Previous work

Theorem (Crovisier-Pujals-Sambarino)

C^1-far from homoclinic tangencies there are finitely many sinks and sources (No Newhouse Phenomena).

Bonatti and Diaz showed that when tangencies are present (in dim $M \geq 3$) one may have Newhouse phenomena.

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IMPORTANT REMARK: If quasi-attractors accumulate, their Hausdorff limit is a partially hyperbolic set as in our hypothesis. This motivates the study of such sets.

Main Technical Theorem

Theorem (Joint with S.Crovisier and M.Sambarino)

For a diffeomorphism f in an open and dense subset of $\text{Diff}^1(M)$ *if Λ is a W^u-saturated partially hyperbolic set then Λ displays a "(uniform) form of non joint integrability of $E^s \oplus E^u$".*

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Related to open and denseness of accesibility (Dolgopyat-Wilkinson) but harder, since it controls non-integrability in proper subsets. Even if the whole M is partially hyperbolic, the result does not follow from density of stable accesibility.

Geometry of partially hyperbolic W^u-saturated sets also appears in work of Crovisier-Pujals and Abdenur-Crovisier.

Idea of the proof

Some Consequences

Consequence 1

If f belongs to an open and dense subset of $\mathrm{Diff}^1(M)$ *and* Λ *is a* W^u*-saturated compact partially hyperbolic set with* $\dim E^c = 1$ *then* Λ *contains finitely many minimal* W^u*-saturated sets.*

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Consequence 2

In an open and dense subset of $\text{Diff}^1(M)$ *far from homoclinic tangencies for* M *of dimension 3 there are at most finitely many quasi-attractors.*

Some ideas

Thanks!