

When do two nilpotent matrices commute?

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Abstract

Let $B \in \text{Mat}_n(\mathbf{k})$ be an $n \times n$ nilpotent matrix with entries in the field \mathbf{k} and denote by P the partition $P \vdash n$ determined by the sizes of the blocks of the Jordan block matrix J_P conjugate to B . We call P the *Jordan type* of B . Consider the centralizer \mathcal{C}_B and the subscheme $\mathcal{N}_B \subset \mathcal{C}_B$ comprised of its nilpotent elements. Since \mathcal{N}_B is irreducible, there is a Jordan type $Q(P)$ for $A \in \mathcal{N}(B)$ that is maximum in the Bruhat order on partitions of n . What is $Q(P)$? Polona Oblak determined the largest part of $Q(P)$ and made a recursive conjecture for $Q(P)$. We discuss results on $Q(P)$ by several groups, and a proof of "half" the conjecture using a poset attached to \mathcal{N}_B . We also discuss connections with the punctual Hilbert scheme. This is joint work with Leila Khatami.