Models of structured populations, their analysis and application to cell differentiation problems

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The lectures are devoted to the applications of differential equations models to describe processes of stem cell self-renewal and differentiation. We refer to the hematopoietic system, although the models and their analysis can be also extended to the other stem cell systems. After reviewing some existing models of hematopoiesis and current challenges, we focus on structured population models, which are first order transport equations and may be additionally coupled with the other differential equations. We discuss basic features of structured population models and methods for their analysis. The method of relative entropy has proven to be a very useful tool to investigate the convergence of solutions to steady states. The idea consists in using qualitative arguments such as strictly convex entropies to provide convergence. Another method is to reduce the system to a delay differential equation. We explore the applicability of these and other methods on the examples of structured population models of hematopoiesis. In addition, we show the links between the structured models and the other approaches to model hematopoiesis, such as multi-compartmental models or agent-based models.

Main references:

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3. A. Marciniak-Czochra, T. Stiehl, A. D. Ho, W. Jaeger and W. Wagner (2008) Modeling asymmetric cell division in hematopietic stem cells regulation of self-renewal is essential for efficient repopulation. *Stem Cells Dev.* 17: 1-10.

4. P S Kim, P P Lee, and D Levy. Modeling imatinib-treated chronic myelogenous leukemia: reducing the complexity of agent-based models. Bull Math Biol., 70(3):728 44, 2008.

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