

Yoshi Kohayakawa. The number of B_h -sets of a given cardinality.

Abstract. Let A be a set of integers. For any integer $h \geq 2$, we say that A is a B_h -set if all the h -wise sums

$$a_1 + \cdots + a_h \quad (a_1, \dots, a_h \in A, a_1 \leq \cdots \leq a_h)$$

are distinct. Let $[n] = \{1, \dots, n\}$. A natural question is the determination or estimation of the extremal function

$$F_h(n) = \max\{|A| : A \subset [n] \text{ is a } B_h\text{-set}\}.$$

The particular case of this problem in which $h = 2$ was raised by Simon Sidon and, therefore, B_2 -sets are known as *Sidon sets*. An immediate argument shows that $F_h(n) = o(n)$ and, therefore, this is a so called ‘degenerate’ extremal problem. As it is often the case with degenerate problems, the structure of the *extremal*, that is, largest, B_h -sets $A \subset [n]$ is not well understood.

We address the simpler problem of estimating the *number* of B_h -sets $A \subset [n]$ of a given cardinality. As a consequence of these bounds, we determine, asymptotically, for any integer $m \leq n$, the cardinality of the largest B_h -sets contained in a typical m -element subset of $[n]$. This is joint work with D. Dellamonica Jr, S.J. Lee, V. Rödl and W. Samotij.
