KAM for PDES

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Abstract:
The KAM theorem, after Kolmogorov, Arnold and Moser is fundamental in many areas of mathematics (dynamical systems, differential and complex geometry, for instance). The basic idea is to prove a persistence result for quasi-periodic motions for Hamiltonian systems. Originally, the theorem has been developed in a finite dimensional framework. In recent decades, major progresses have been done in the direction of infinite dimensional systems, such as lattices or PDEs.

In this series of lectures, I will describe a new method, quite flexible and based on geometric cancelations, allowing to construct invariant tori for infinite dimensional systems with the final purpose to apply the strategy to strongly ill-posed PDEs coming from fluid dynamics.

The outline of the course will be: first, I will state and prove the standard KAM theorem in finite dimensions (after recalling some basic facts of symplectic geometry and Hamiltonian systems). Second, I will go on the case of lattices, constructing finite and infinite dimensional invariant tori. Finally, I will describe the KAM iteration for PDEs, where several issues intrinsic to PDEs arise.