Yann Bugeaud. Around the Littlewood conjecture.
Abstract. The Littlewood conjecture in diophantine approximations claims that every pair $(\alpha, \beta)$ of real numbers satisfies

$$
\inf _{q \geq 1} q\|q \alpha\|\|q \beta\|=0
$$

where $\|\cdot\|$ denotes the distance to the nearest integer. In 2004, de Mathan and Teulié asked the following analogous question: for a given prime number $p$, is it true that

$$
\left.\inf _{q \geq 1} q\|q \alpha\|| | q\right|_{p}=0
$$

holds for every real number $\alpha$ ? Here, $|\cdot|_{p}$ denotes the $p$-adic absolute value normalized such that $|p|_{p}=p^{-1}$. We present recent results towards the resolution of these two problems, which are still not solved.

