Description of Attainable Sets of Differential Inclusions through Optimal Control*

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Abstract

The importance and applicability of control systems are well known. It is also well known that control systems can be modeled as differential inclusions.

The attainable set of a differential inclusion is the set of all possible system states at time t. One of the main problems of the theory of differential inclusions consists in describing and estimating their attainable sets. It should be noted that the exact description of attainable sets is a difficult problem even in the case of linear dynamics. A procedure for estimating the attainable set is quite complicated compared to the numerical methods for differential equations.

The estimation theory and related algorithms basing on ideas of constructing outer and inner set-valued estimates of attainable sets have been developed. For instance, using the advantages of ellipsoidal calculus, two-sided approximations using parallelotopes, polytopic approximation methods, the "so called" exponential formula, among others.

This work addresses an alternative approach, based on an optimal control tool, to obtain a description of the attainable sets of differential inclusions. A complete description of the attainable sets are possible for differential inclusions involving nonlinear systems which are affine in the control variable. Systems that are nonlinear on both the state and the control variables are also treated. In this case, approximate reachable sets are obtained.

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