Continuous Numerical Solution of a Differential Equation

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I propose an algorithm to continuously approximate the solution of an ordinary differential equation and perhaps, in future work, to a partial differential equation. The approximate solution is a spline whose degree is a parameter and whose coefficients are determined by solving a system of polynomial equations. The latter is obtained by taking the partial derivatives of a Lagrangian function, which represents an optimization problem closely related to the differential equation.

A comparison with existing procedures like collocation methods is in order. Additionally some propositions derived by the numerical results need to be prove or determine the conditions under which they are true. One proposition is that the linearity of the system of polynomial equations depends on the linearity of the differential equation. Another proposition or theorem is that this system has a unique real solution provided the differential equation has a unique solution. Finally, the last proposition is that the sequence of splines uniformly converges to the exact solution of the differential equation when its degree is taking to infinity or when the length of the subintervals defining the piecewise polynomial decreases to almost zero.

The main contribution is the precision to approximate the exact solution of the differential equation in every point, which can compete with the most actual accurate methods. Most importantly, this method provides a measure for the approximate solution to satisfy the differential equation, which hopefully can be related with the distance of the spline to the exact solution.

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