Solving feasibility problems with complementarity

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Problems with complementarity constraints appears frequently in optimization and since it is related to the notion of system equilibrium, it has significant applications in engineering, economics and sciences. First of all, note that Mathematical Programming problems with complementarity constraints may be expressed, perhaps after some reformulation, in the form:

$$\begin{array}{rl} \text{Minimize} & f(x,y,w) \\ s.t. & h(x,y,w) = 0 \\ & \min\{x,w\} = 0 \end{array} \tag{1}$$

where $x, w \in \mathbb{R}^n$, $y \in \mathbb{R}^m$, $p \geqslant 1$ and $h \in \mathbb{R}^{p-1}$, for all x, y, w.

We denote by $\min\{x,w\}$ the vector $(\min\{x_1,w_1\},...,\min\{x_n,w_n\})^T$. The complementarity constraints, $\min\{x,w\} = 0$, may be expressed in many different ways, we choose to use one of the most popular, namely, $x_iw_i = 0$, $x_i \geqslant 0$, $w_i \geqslant 0$, $i = 1,...,n$.

Trying to solve (1) by means of standard nonlinear programming problems presents some difficulties, including solutions that do not satisfies the KKT conditions and availability of derivatives. The first one is a consequence of the "double zeros" points, in other words, those points which for certain constraint $x_iw_i = 0$ follows that $x_i = w_i = 0$. The second one is related with the constraints of (1) that involves the KKT conditions of one or many lower level problems, as an example, we have the case when they represent a Bilevel Problem or a Nash-Equilibrium problem.

We emphasize that in many practical cases we are most concerned in obtaining reasonably low values for f, than really minimize the objective function. Thus, we reduce the problem (1) to the feasibility problem of finding x, y, w, t such that:

$$f(x,y,w) + t^2 = c_t, \quad h(x,y,w) = 0, \quad x \geqslant 0, \quad w \geqslant 0 \;\; and \;\; x^Tw = 0, \tag{2}$$

where c_t represents a target to be achieved and t is a slack variable.

In view of the above, we present an algorithm based in [1] to solve (2), that generate a feasible sequence, uses non-monotone line-searches and combine the Newton Method and the Projected Gradient Method. In addition, we test the algorithm to solve many problems of MacMEPC collection [3], comparing the numerical results with the Projected Levemberg-Marquardt Method [2].

References

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