Minimal graphs over unbounded domains of Hadamard manifolds

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Resumo/Abstract:

Given a Hadamard manifold M with sectional curvature $K_M \leq -1$, the existence of solution of the Dirichlet problem for minimal hypersurface equation over an unbounded domain $\Omega \subseteq M$ is investigated. The boundary data is prescribed on its ordinary boundary $\partial\Omega$ and on its asymptotic boundary $\partial_\infty\Omega$. It is then proved that if Ω is "strictly convex" at infinity and mean convex, then solutions exist for any continuous (with respect to the cone topology) boundary data. Moreover, mean convexity of Ω is a necessary condition.

The previous result seams to be useful to answer the following question: under what conditions the entire Hadamard manifold M admits solutions to the asymptotic Dirichlet problem for minimal hypersurface equation, for any boundary data $\psi \in C^0(\partial_\infty M)$? Some known examples prove that the condition $K_M \leq -1$ is not sufficient; on the other hand, if in addition strict convexity of M at infinity is required, then existence is proved. It is then conjectured that, in analogy with the bounded case, some kind of convexity of M is necessary.

References

[1] M. Telichevesky , *A note on minimal graphs over certain unbounded domains of Hadamard manifolds* , Pacific Journal of Mathematics, 281 (1), 2016. 243–255.