Large values of the zeta-function at its critical points

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Let $\rho_1 = \beta_1 + i\gamma_1$ denote a zero of $\zeta'(s)$. The question of how large the zeta-function can be at such a point has relevance to the geometry of the level curves of the zeta-function. We show that for all T sufficiently large there exists a zero ρ_1 of $\zeta'(s)$ with β_1>1 and T< γ_1<2T such that

$|\zeta(\rho_1)| >$ (1/4 $e^{C_0} - o(1)$) log log T,

where C_0 is Euler's constant. This is unconditional but, assuming the Riemann Hypothesis, we show that for all T sufficiently large there also exists a zero ρ_1 with β_1>1 and T< γ_1<2T such that

$|\zeta(\rho_1)| <$ (1/2 $e^{C_0} + O(1)$) log log T.

We conjecture that this holds with 1/2 replaced by 1/4. This is joint work with Hugh Montgomery.