PERTURBED DIFFERENTIAL EQUATION AND REGULARIZATION BY NOISE

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Resumo/Abstract:
Paths of some stochastic processes such as fractional Brownian motion have some amazing regularizing properties. It is well known that in order to have uniqueness in differential systems such as
\[ dy_t = b(y_t)dt \]
b needs to be quite regular. As soon as the last equation is perturbed by a suitable stochastic process \( w \), the oscillations of such a process will guarantee that the following system
\[ y_t = x + \int_0^t b(y_r)dr + w_t \]  
(1)
has a unique solution for really irregular \( b \).

After recalling some basic facts we will show that the study of the following stochastic averaging operator
\[ T^w_t b(x) = \int_0^t b(x + w_r)dr \]
will allow us to solve equation (1) for \( b \) on which we have some control on the growth of the Fourier transform. This will allow us to extend such equations when \( b \) are not functions but distributions.

As an application, we will show that the stochastic transport equation driven by fractional Brownian motion with \( H \in (0,1) \)
\[ \partial_t u + b.\nabla u + \nabla u.dB^H_t = 0 \]
has a unique solution when \( u_0 \in L^\infty \) and \( b \) is a possibly random \( \alpha \)-Hölder continuous function for \( \alpha \) large enough.

This is a joint work with Massimiliano Gubinelli.