# PERTURBED DIFFERENTIAL EQUATION AND REGULARIZATION BY NOISE

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**Resumo/Abstract:**
Paths of some stochastic processes such as fractional Brownian motion have some amazing regularizing properties. It is well known that in order to have uniqueness in differential systems such as

$$\mathrm{d}y_t = b(y_t)\mathrm{d}t$$

$b$ needs to be quite regular. As soon as the last equation is perturbed by a suitable stochastic process $w$, the oscillations of such a process will guarantee that the following system

$$y_t = x + \int_0^t b(y_r)\mathrm{d}r + w_t \tag{1}$$

has a unique solution for really irregular $b$.

After recalling some basic facts we will show that the study of the following stochastic averaging operator

$$T_t^w b(x) = \int_0^t b(x + w_r)\mathrm{d}r$$

will allow us to solve equation (1) for $b$ on which we have some control on the growth of the Fourier transform. This will allow us to extend such equations when $b$ are not functions but distributions.

As an application, we will show that the stochastic transport equation driven by fractional Brownian motion with $H \in (0,1)$

$$\partial_t u + b.\nabla u + \nabla u.\mathrm{d}B_t^H = 0$$

has a unique solution when $u_0 \in L^\infty$ and $b$ is a possibly random $\alpha$-Hölder continuous function for $\alpha$ large enough.

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*This is a joint work with* Massimiliano Gubinelli.